KEY CONCEPT

1. DEFINITION

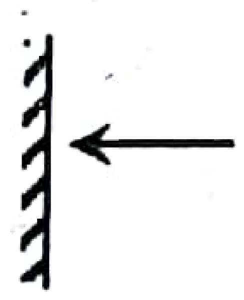
When a light ray strikes the surface separating two media, a part of it gets reflected, i.e., returns back in the initial medium. It is known as reflection.

2. TERMS RELATED TO REFLECTION

2.1 RAY :

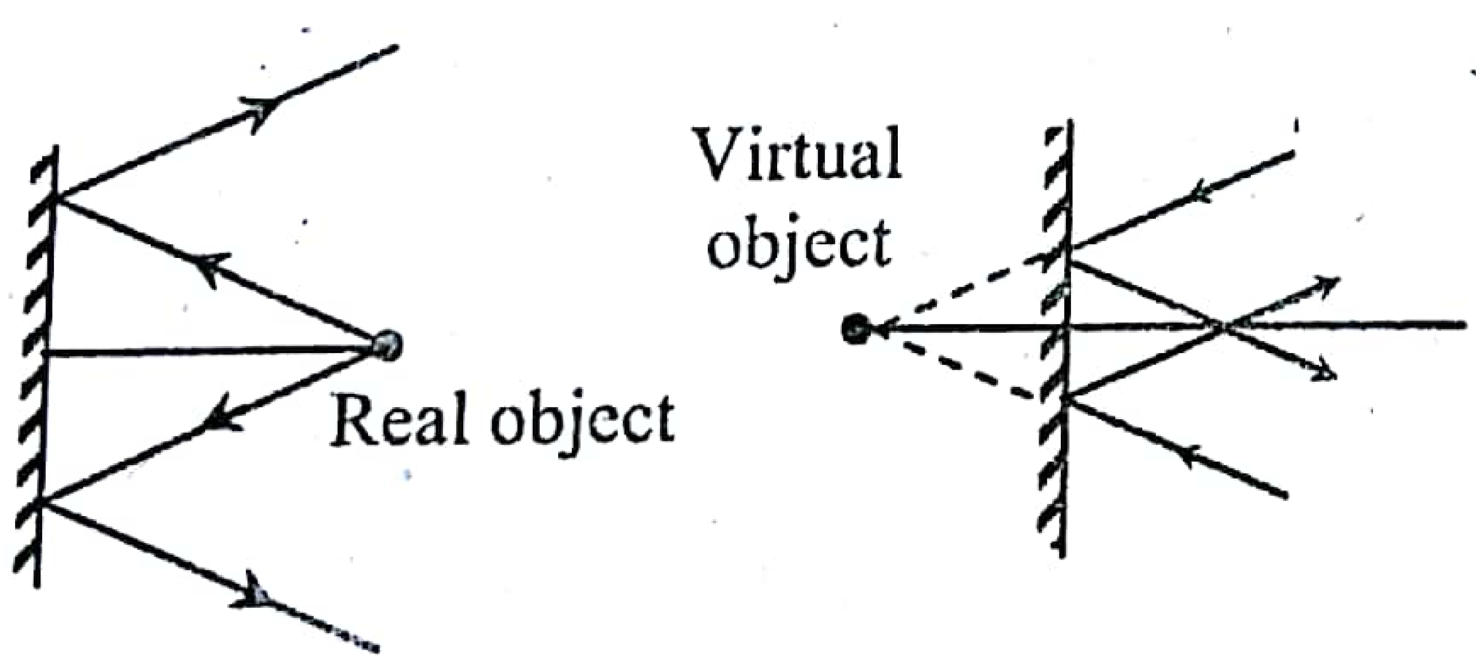
A ray of light is the straight line path of transfer of light energy. It is represented by a straight line with an arrow - head indicating the direction of propagation.

2.2 MIRROR :



It is a highly polished smooth surface from which most of the incident light gets reflected. It is represented by a line with hatches in the reverse side of the smooth surface.

2.3 OBJECT :

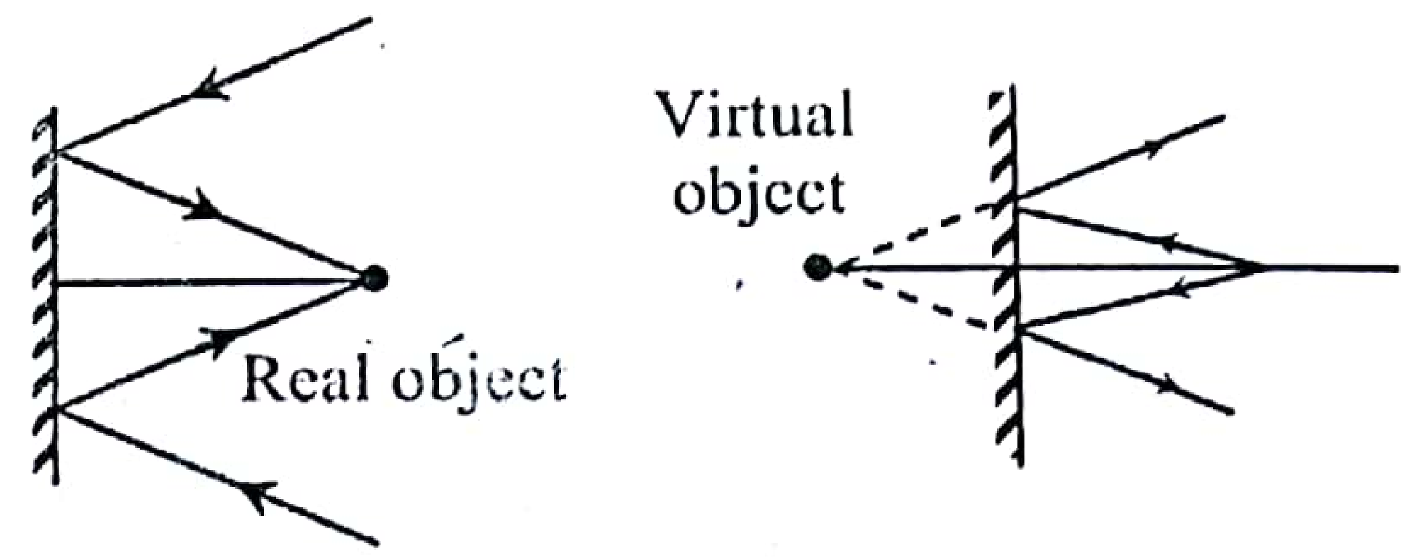


A. Point from which incident ray actually diverge is called real object. Or point at which incident rays appear to converge is called virtual object.

B. object is defined on the basis of incident ray.

C. Minimum two rays are required to show the position of object

2.4 IMAGE :



A. Point at which reflected or refracted rays actually converge is called real image. Or point from which reflected or refracted rays appear to diverge is called virtual image.

B. Minimum two reflected or refracted rays are required to determine the image position.

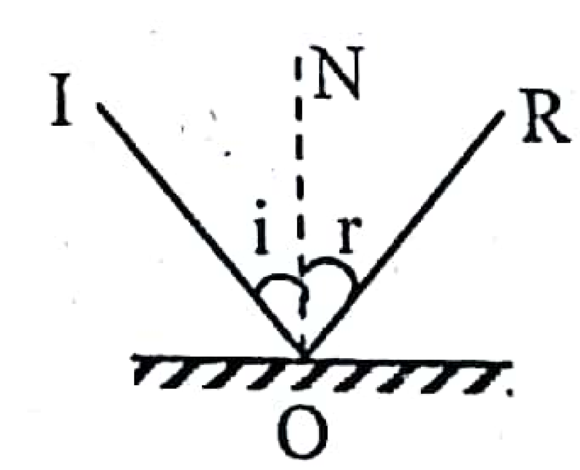
3. BASIC LAWS

3.1. Law Of Rectilinear Propagation Of Light states that light propagates in straight lines in homogeneous media.

3.2. Law Of Independence Of Light Rays states that rays do not disturb each other upon intersection.

3.3. Law Of Reversibility Of Light Rays states that rays retrace their path when their direction is reversed.

3.4. Laws Of Reflection :

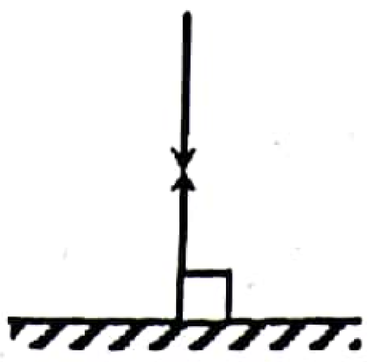


A. The incident - ray, reflected ray and normal to the reflecting surface at the point of incidence all lie in the same plane.

B. The angle of reflection is equal to the angle of incidence, i.e. $\angle i = \angle r$

Note :

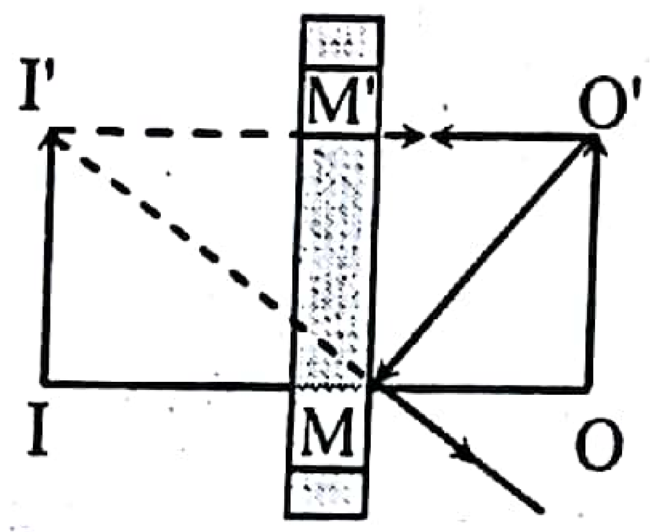
1. These two laws are also valid for curved surfaces.



2. If $\angle i = 0$ then $\angle r = 0$, i.e., if a ray is incident normally on a boundary, after reflection it retraces its path.
3. None of frequency, wavelength and speed changes. However intensity and hence amplitude usually decreases since $I \propto A^2$
4. There is a phase change of π if reflection takes place from denser medium.

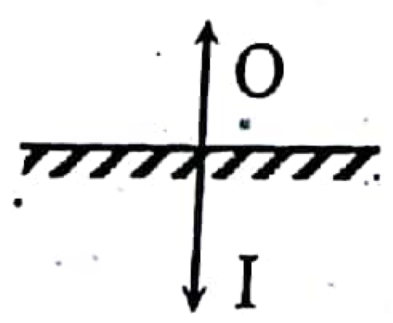
4. CHARACTERISTICS OF REFLECTION AT PLANE MIRROR

4.1. The image is always of same size and at same distance behind the mirror as the object in front of it.

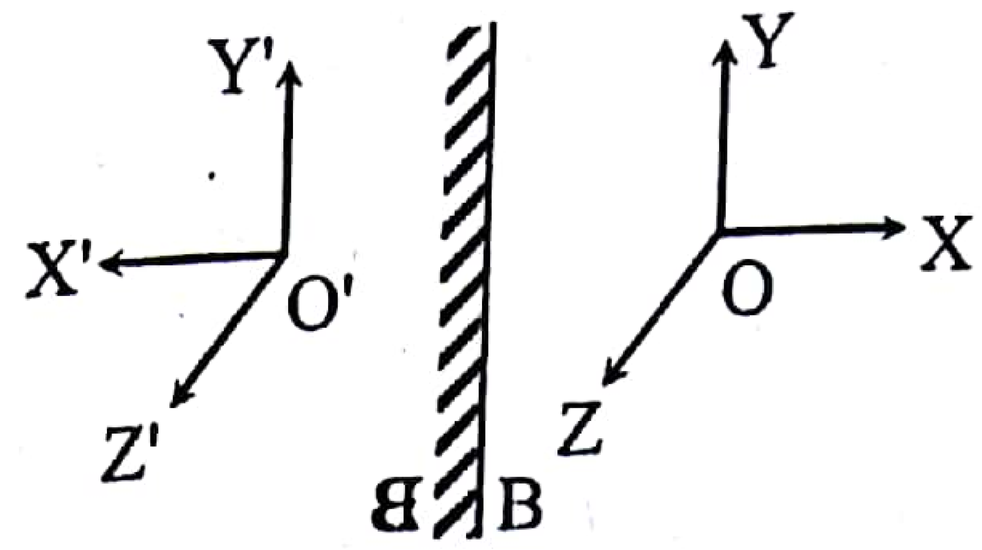


4.2. When the object is real virtual image is formed and vice - versa.

4.3. The image is always erect when the object is in front of the mirror but inverted when the object is above the mirror.



4.4. The image is laterally inverted. The mirror actually reverses front and back in three dimensions (and not left to right) i.e. only x-direction is reversed resulting in the change of left into right or vice - versa. A plane mirror changes right handed co-ordinate system to left handed one.

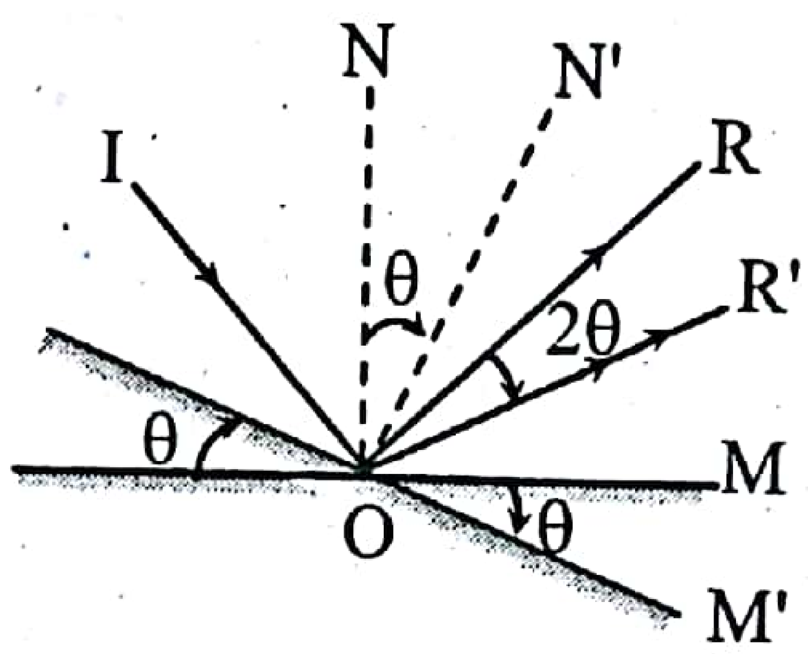


4.5. As every part of mirror forms complete image of an extended object and due to superposition of images brightness will depend on the light reflecting area of the mirror. [It can be explained by Haygen's principle.]

4.6. Though every part of a mirror forms complete image of an object, we usually see only that part of image from which light after reflection from the mirror reaches our eye.

4.7. Deviation δ is defined as the angle between directions of incident ray and emergent ray. So if light is incident at an angle of incidence i, $\delta = \pi - (i + r) = \pi - 2i$

4.8. If keeping the incident ray fixed, the mirror is rotated by an angle θ, about an axis in the plane of mirror, The reflected ray is rotated through an angle 2θ.



4.9 Most of the smooth surfaces works as a good reflector when light falls at grazing incidence ($\angle i \to 90°$)

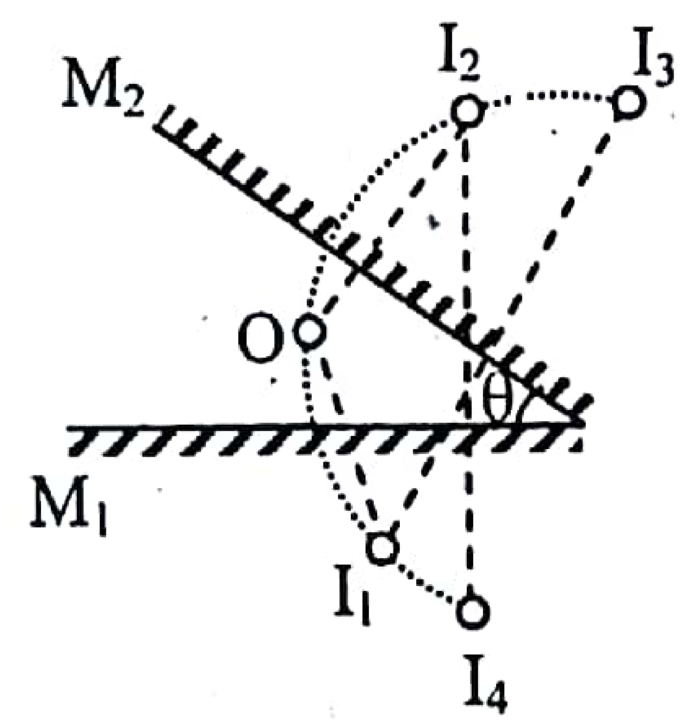
4.10 From smooth reflecting surface we get parallel beams of light which produces glare. However from rough surface we get diffused radiation.

4.11 If object moves towards the plane mirror at speed v, the image moves towards the plane mirror at speed – v. So, the image speed w.r.t object is $-v - v = -2v$

4.12 If mirror is moved towards (or away from) the object with speed v the image will move towards (or away from) the object with a speed 2v.

5. IMAGES FROMED BY TWO MIRRORS

1. When two plane mirrors are inclined at an angle q and an object is placed in between them due to multiple reflection more than one images are formed. This number of image n is either



$\frac{360°}{\theta}$ or $\left(\frac{360°}{\theta} - 1\right)$

accordingly as $\frac{360°}{\theta}$ is odd or even respectively.

Again if $\frac{360°}{\theta}$ is odd and the object is place symmetrically between two mirrors, then final two images coincide and there by leaving $\left(\frac{360°}{\theta} - 1\right)$ images.

$n = 360°/\theta$	Position Of Object	Number Of Images
even	anywhere	n – 1
odd	symmetric	n – 1
	assymetric	n

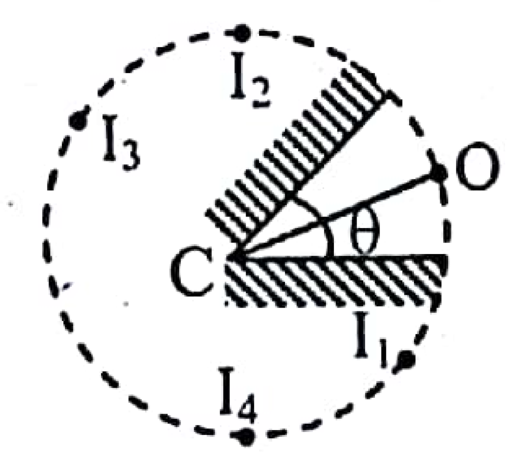
Note :

1. If an object is placed between two parallel mirrors, $\theta = 0°$, the number of images formed is but of decreasing intensity in accordance with $I \propto (1/r^2)$

2. No. of images for some specific angles :

No.	θ in degrees	n = 360°/θ	No. of images formed when the object is placed	
			symmetrically	Assymmetrically
1	0	∞	∞	∞
2	30°	12	11	11
3	45	8	7	7
4	60	6	5	5
5	72	5	4	5
6	90	4	3	3
7	120	3	2	3

2. All the images lie on a circle whose radius is equal to the distance between object O and the point of intersection of mirrors C.



3. If θ is given, n is unique but if n is given, q is not unique. Since same number of images can be formed for different θ.

4. The number of images seen may be different from number of images formed and depends on the position of the observer relative to object and mirrors.

6. CURVED MIRRORS

6.1 A curve mirror is a smooth reflecting part (in any shape) of a symmetrical curved surface such as paraboloidal, ellipsoidal, cylindrical or spherical.

Note : Here we will discuss only on spherical mirrors.

6.2 CONCAVE MIRROR :

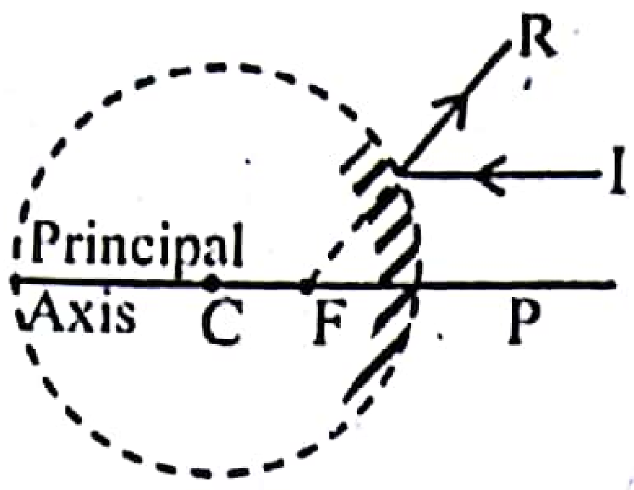
If the reflection takes place from the inner surface of a spherical mirror, then the mirror is called concave mirror.

6.3 CONVEX MIRROR :

If the outer surface of the spherical mirror acts as a reflector then the mirror is called convex mirror.

7. TERMS RELATED TO SPHERICAL MIRROR

7.1 CENTRE OF CURVATURE (C) :

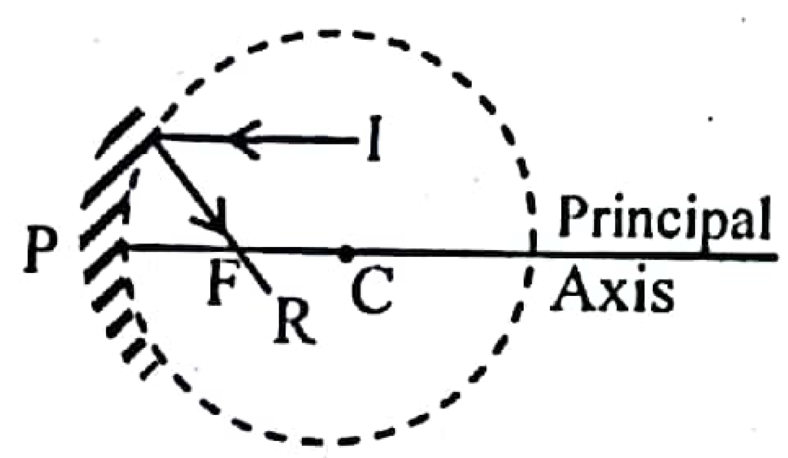


It is the centre of sphere of which the mirror is a part.

7.2 RADIUS OF CURVATURE (R) :

It is the radius of the sphere of which the mirror is a part.

7.3 POLE (P) :



It is the geometrical centre of the spherical reflecting surface.

7.4 PRINCIPAL AXIS :

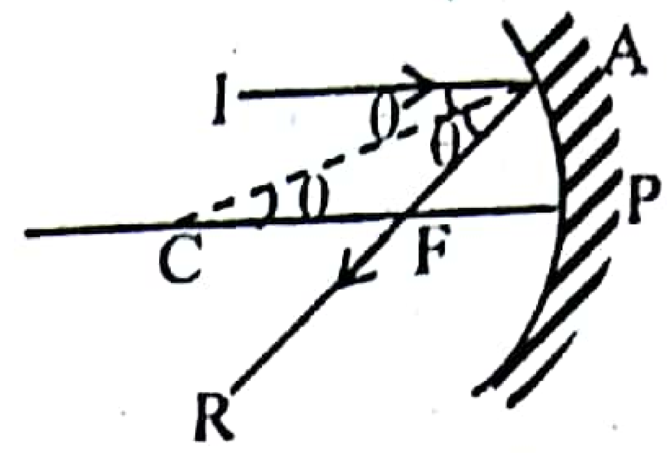
It is the straight line joining the centre of curvature to the pole.

7.5 FOCUS (F) :

When a narrow beam of rays of light, parallel to the principal axis and close to it (known as paraxial rays), is incident on the surface of a mirror, the reflected beam is found to converge (concave mirror) or appear to diverge (convex mirror) from a point on the principal axis. This point is called focus.

7.6 FOCAL LENGTH (F) :

It is the distance between the pole and the principal focus. For spherical mirrors, $f = R/2$



$$\therefore \quad f = f = R\left(1 - \frac{1}{2}\sec\theta\right)$$

θ is different for different rays but for paraxial rays $\theta \rightarrow \Rightarrow \sec\theta \rightarrow 1$

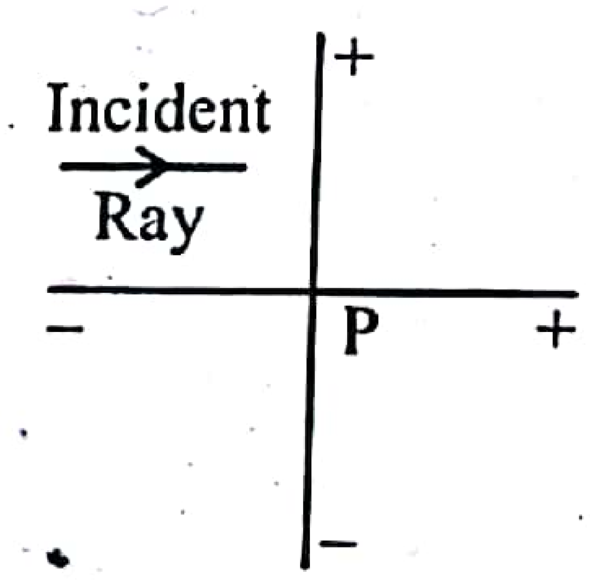
$\therefore f = R/2$

7.7 APERTURE

It is the effective diameter of light reflecting area of the mirror.

8. CARTESIAN SIGN CONVENTION

8.1 All distances are measured from the pole (P) and P is the origin.



8.2 Distances measured to the right of the pole are taken as positive.

8.3 Distances above the principal axis are taken as positive.

8.4 Angles measured from the normal in the anticlockwise sense are positive.

9. MIRROR FORMULAE

9.1 In terms of Cartesian sign convention mirror formula may be expressed as

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Where u, v and f represents object distance, image distance and focal length respectively.

9.2 Linear magnification : Linear magnification is defined as the ratio of the size of image to the size of the object i.e. $\boxed{m = \frac{I}{O}}$

(i) If one dimensional object is placed perpendicular to the principal axis, linear magnification is called transverse or lateral magnification and for mirrors becomes

$$\boxed{m = \frac{I}{O} = \frac{-v}{u}}$$

m = negative, for real object-real image pair, for virtual object-virtual image pair

m = positive, for real-virtual pair

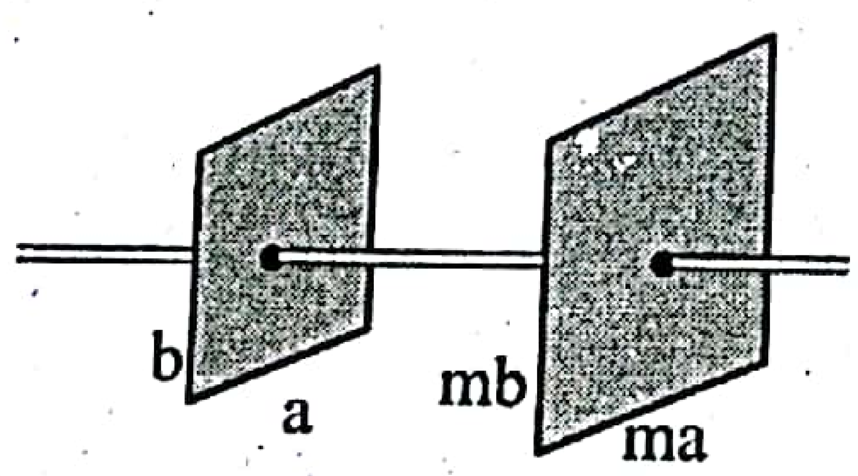
(ii) However, if one dimensional object is placed along the principal axis, the so called longitudinal magnification and becomes

$$m_L = \frac{I}{O} = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$$

for small object

so $$\boxed{m_L = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = m^2}$$

(iii) If a 2-D object is placed with its plane perpendicular to principal axis its magnification called superficial magnification, will be



$$m_S = \frac{\text{area of image}}{\text{area of object}} = \frac{(ma)(mb)}{a \times b} = m^2$$

So $\boxed{m_S = m^2}$

where m → transverse magnification

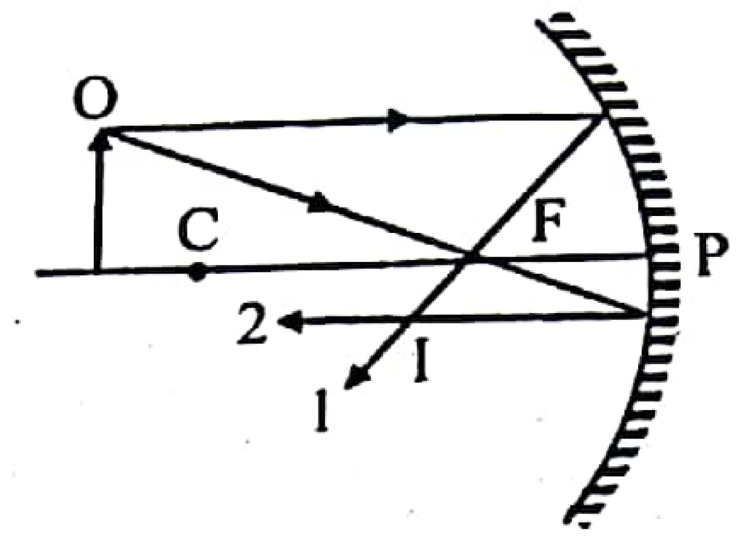
(iv) In case of more than one optical component, the image formed by first component will act as an object for the second and so on. So over all magnification

$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times ....$$

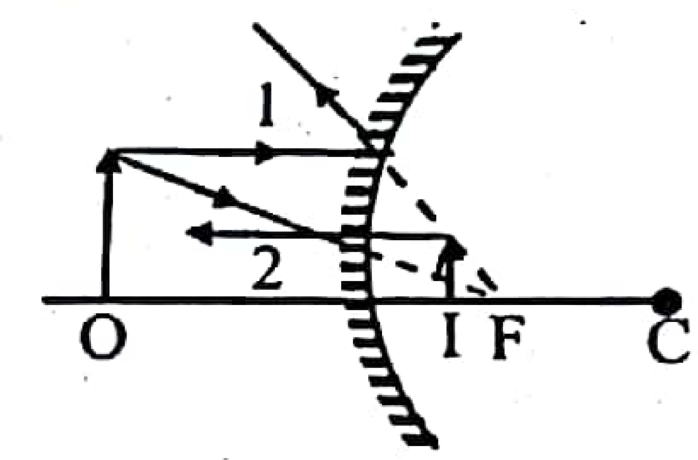
or $$\boxed{m = m_1 \times m_2 \times m_3 \times ....}$$

10. RULES FOR RAY DIAGRAMS

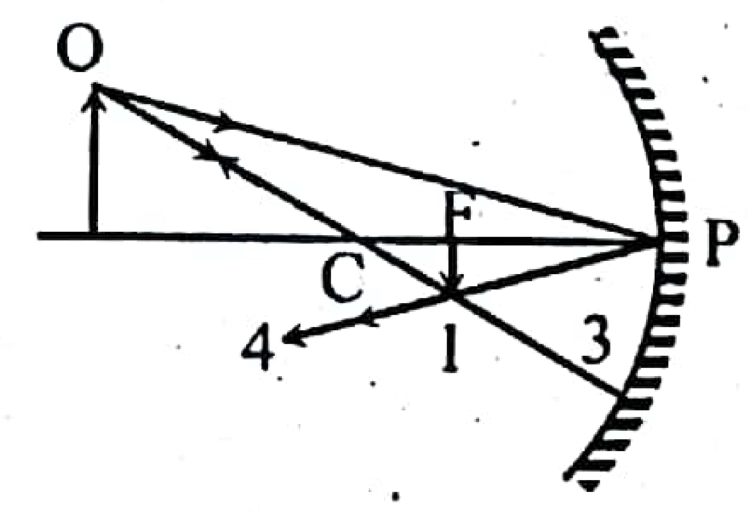
10.1 A ray, initially parallel to the principal axis is reflected through the focus of the mirror.



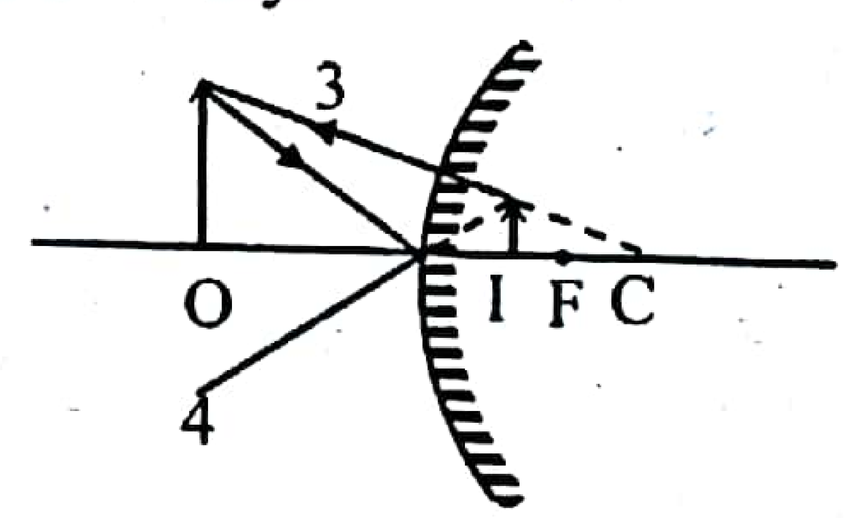
10.2 A ray, initially passing through the focus is reflected parallel to the principal axis.



10.3 A ray passing through the centre of curvature is reflected back along itself.

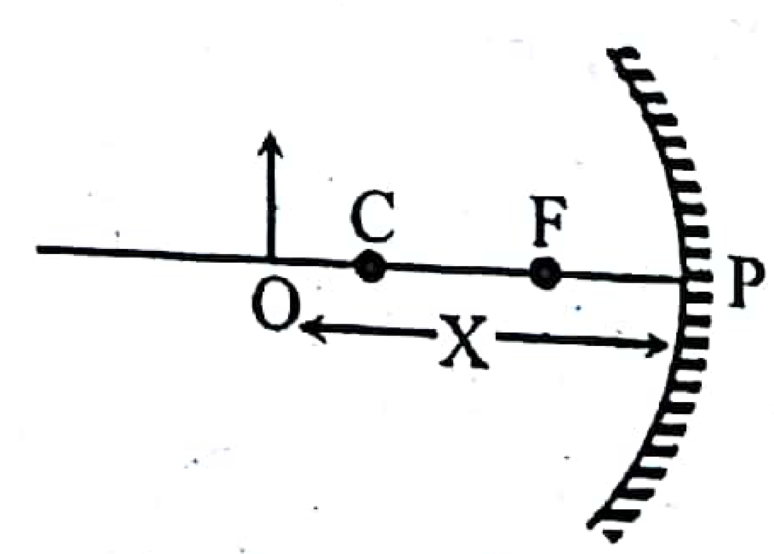


10.4 A ray incident at the pole is reflected symmetrically.



11. POSITIONS, SIZE AND NATURE OF IMAGES

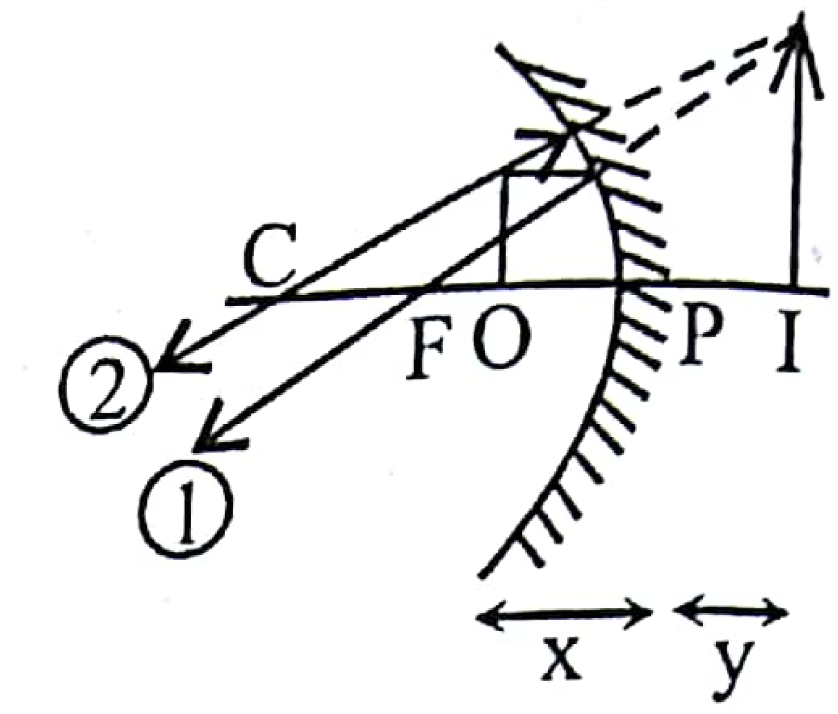
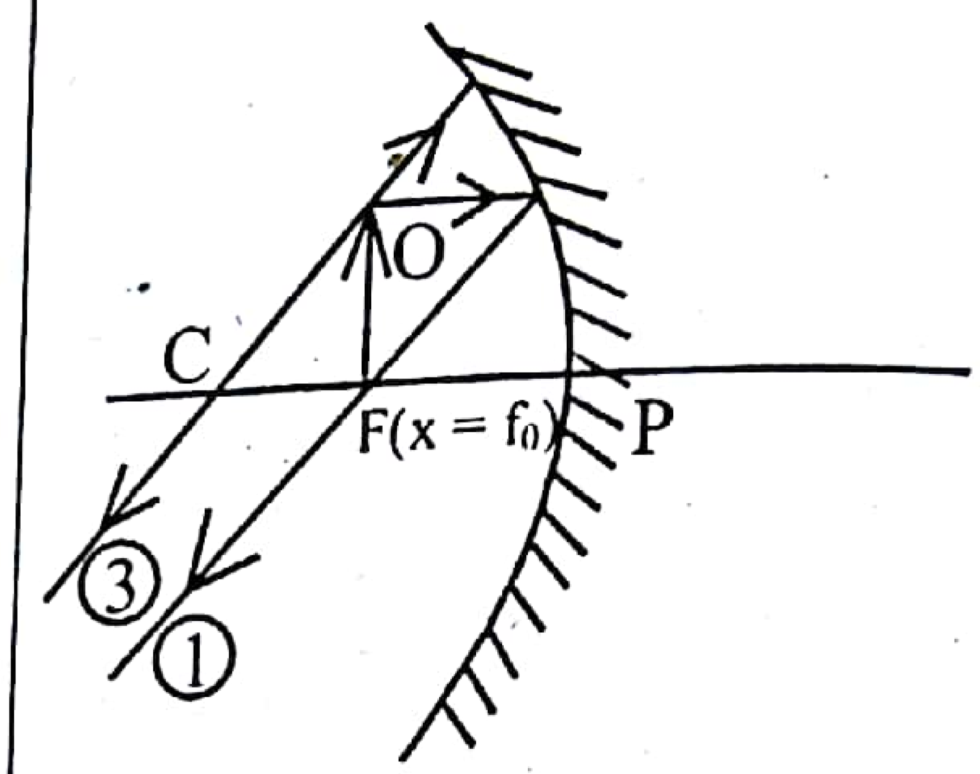
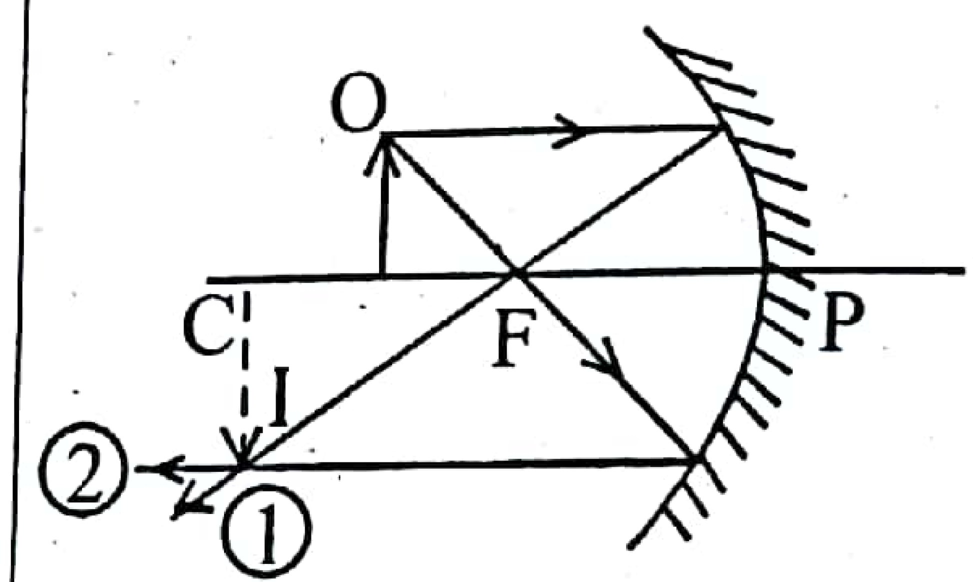
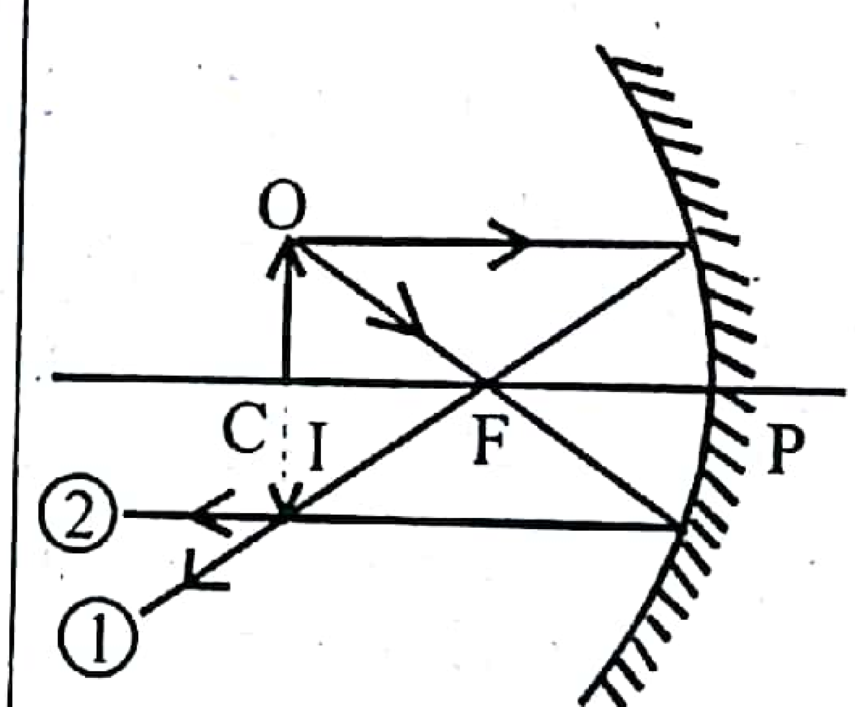
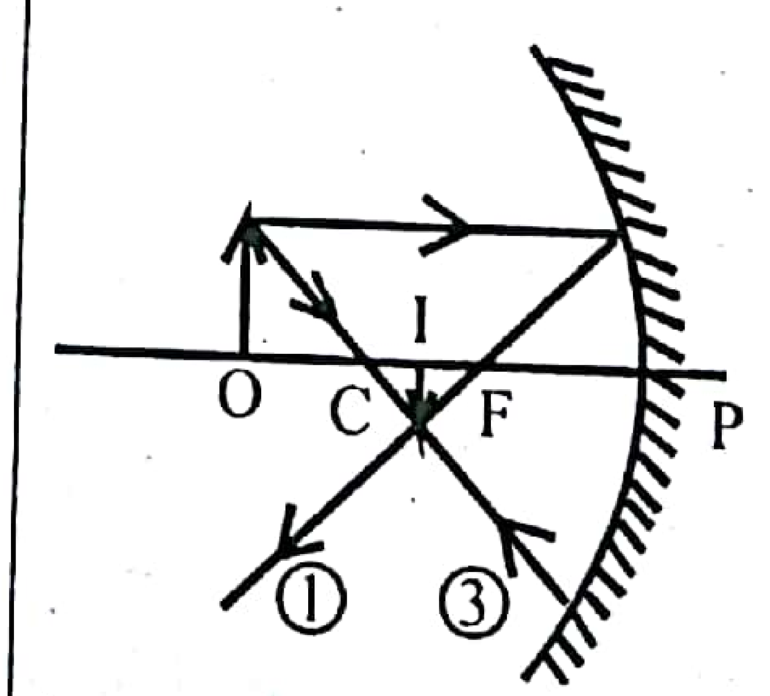
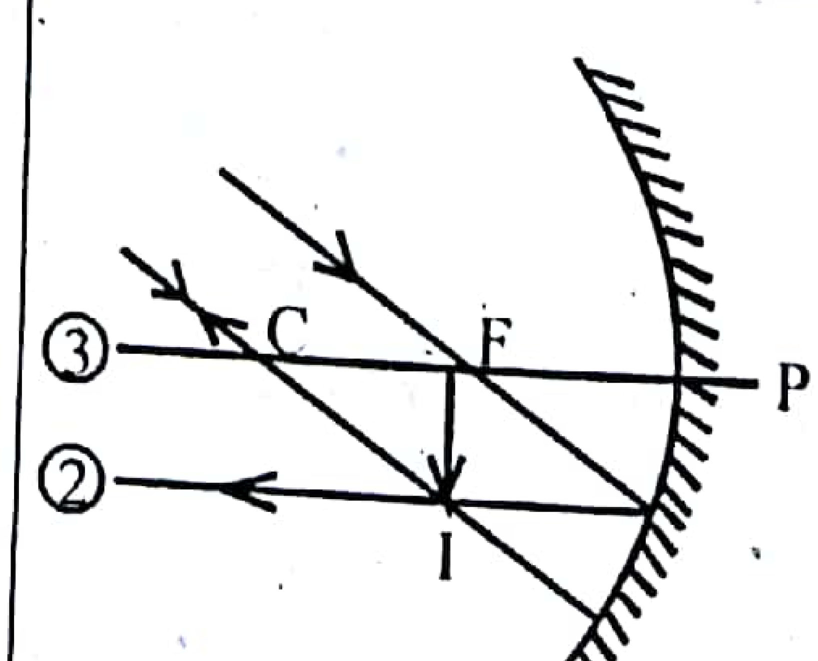
11.1 CONCAVE MIRROR :



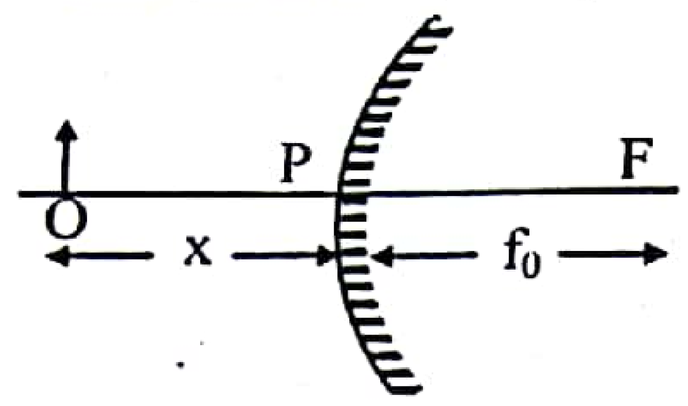
Let, $u = -x$ then

$$\frac{1}{V} + \frac{1}{x} = \frac{1}{-f_0} \Rightarrow v = \frac{xf_0}{f_0 - x}$$

and $$m = \frac{f_0}{f_0 - x}$$

S.No.	Position of Object	Position of Image	Nature	Size	Ray Diagram
1.	Between Pole and Focus	Behind Mirror	Virtual, Erect	Magnified	
2.	At Focus	In front of the Mirror at Infinity	Real, Inverted	Highly Magnified	
3.	Between Focus & centre of curvature	In front of the Mirror, beyond Centre of curvature	Real, Inverted	Magnified	
4.	At Centre of Curvature	At Centre of curvature	Real, Inverted	Equal size	
5.	Between Centre of Curvature and Infinity	Between Focus & Centre of curvature	Real, Inverted	Diminished	
6.	At Infinity	At focus	Real, Inverted	Highly Diminished	

11.2 Convex Mirror



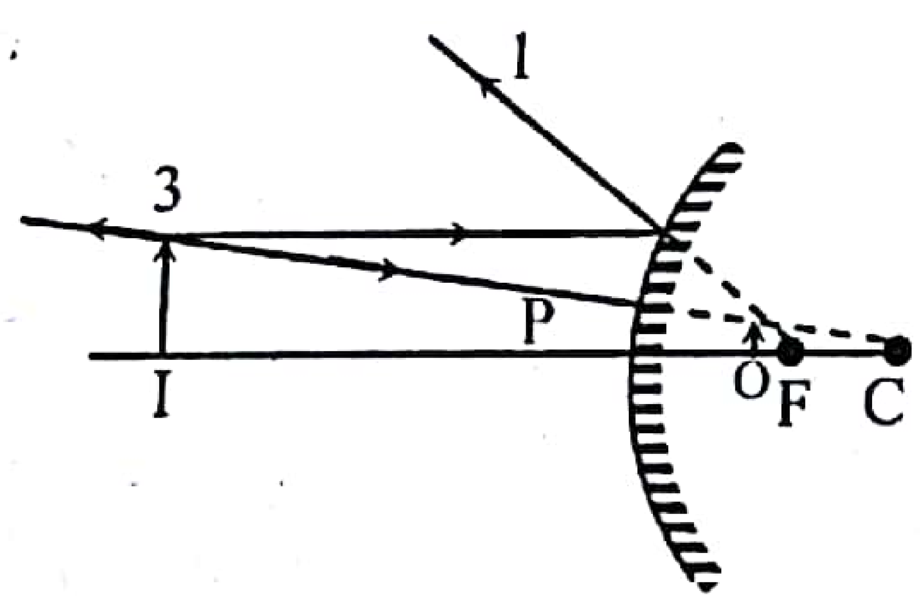
Let $u = -x$

$$\frac{1}{v} + \frac{1}{-x} = \frac{1}{f_0} \Rightarrow v = \frac{xf_0}{f_0 + x} \text{ and } m = \frac{f_0}{f_0 + x}$$

For all position of real object, convex mirror gives virtual, diminished, erect image between pole and focus .

Note :

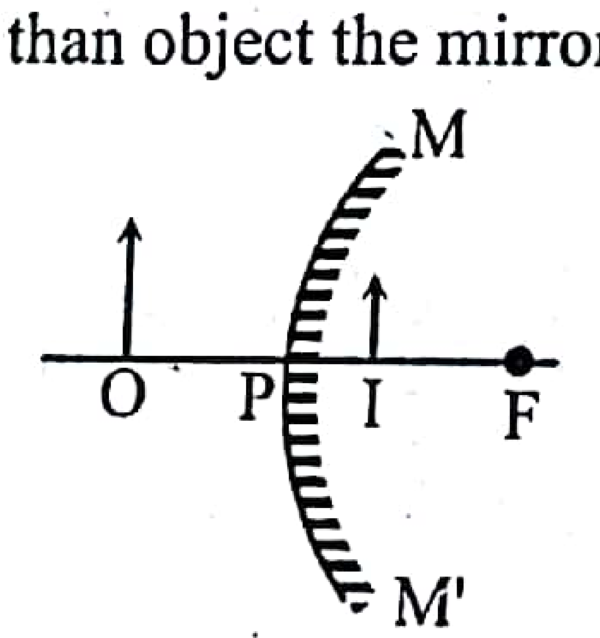
1. For virtual object convex mirror produce real magnified and erect image in front of the mirror if object distance is less than focal length.
2. For virtual object concave mirror produce real, erect- and diminished image in front of the mirror between pole and focus.



IMPORTANT POINTS

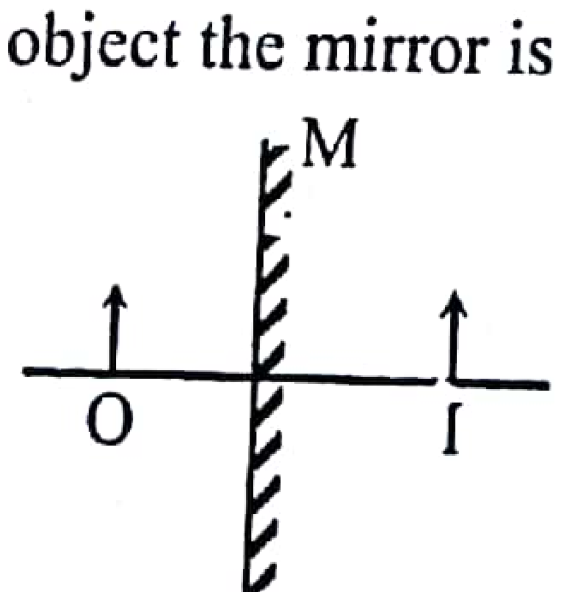
1. For real extended object, if the image formed by a single mirror is erect it is always virtual (i.e., m is +ve) and in this situation if the size of image is :

smaller than object the mirror is convex



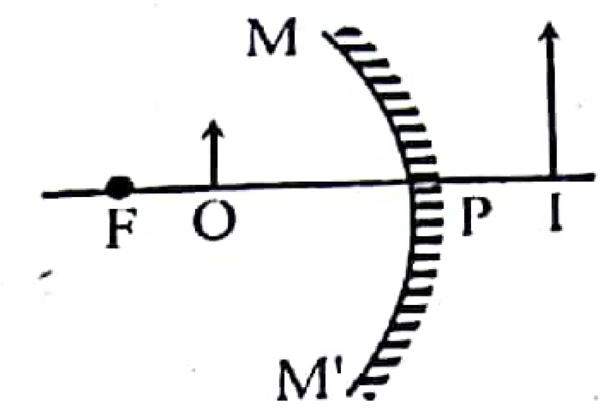
$m < +1$

equal to object the mirror is plane



$m = +1$

larger than object the mirror is concave

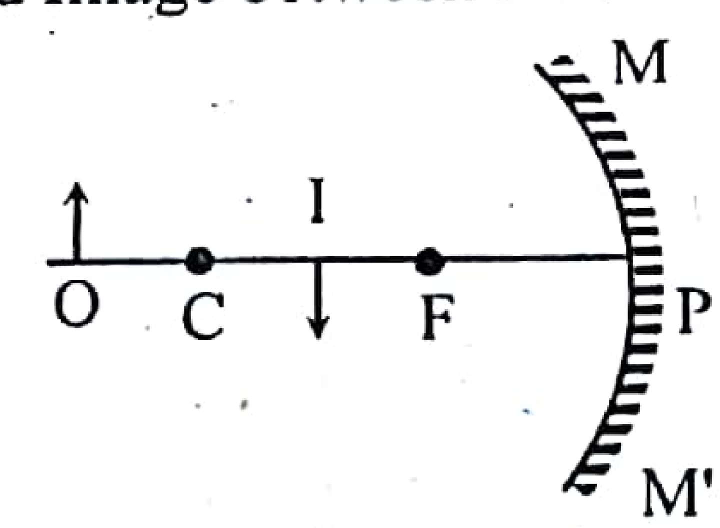


$m > +1$

So by observing the size of erect image in a mirror we can decide the nature of the mirror, i.e. whether it is convex, concave or plane.

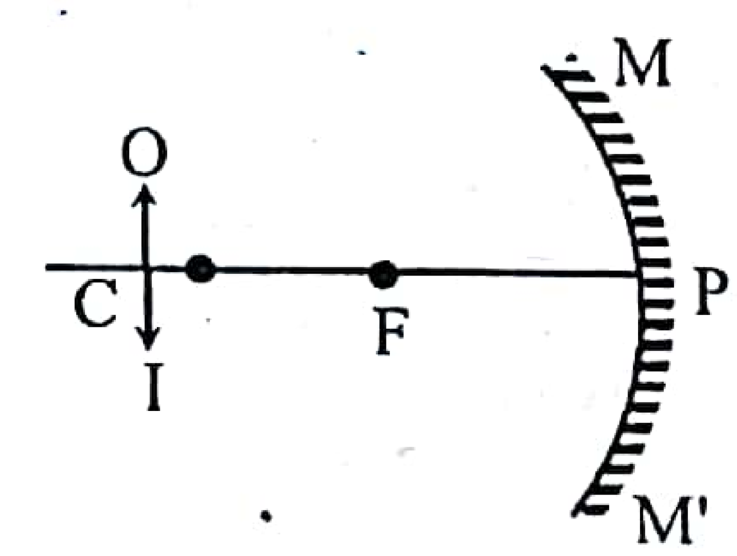
2. For real extended object, if the image formed by a single mirror is inverted, it is always real (i.e., m is –ve), the mirror is concave. In this situation if the size of image is :

Smaller than object, object is between ∞ and C and Image between F and C



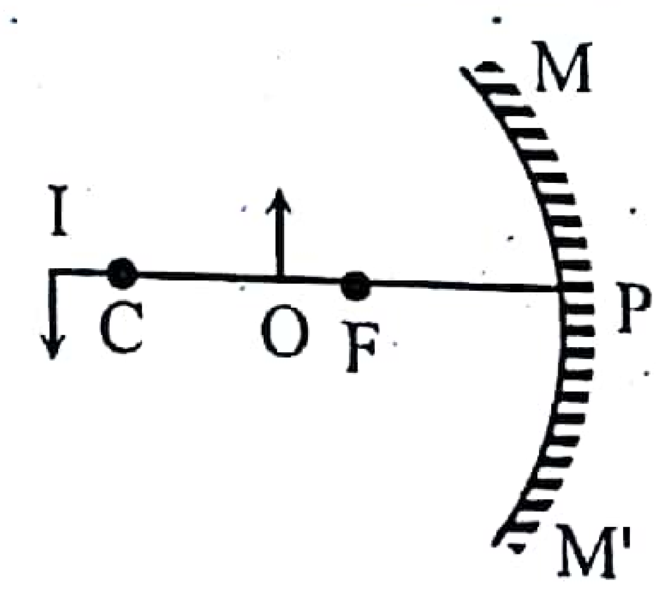
$m < -1$

Equal to object, object is at C and image is at C



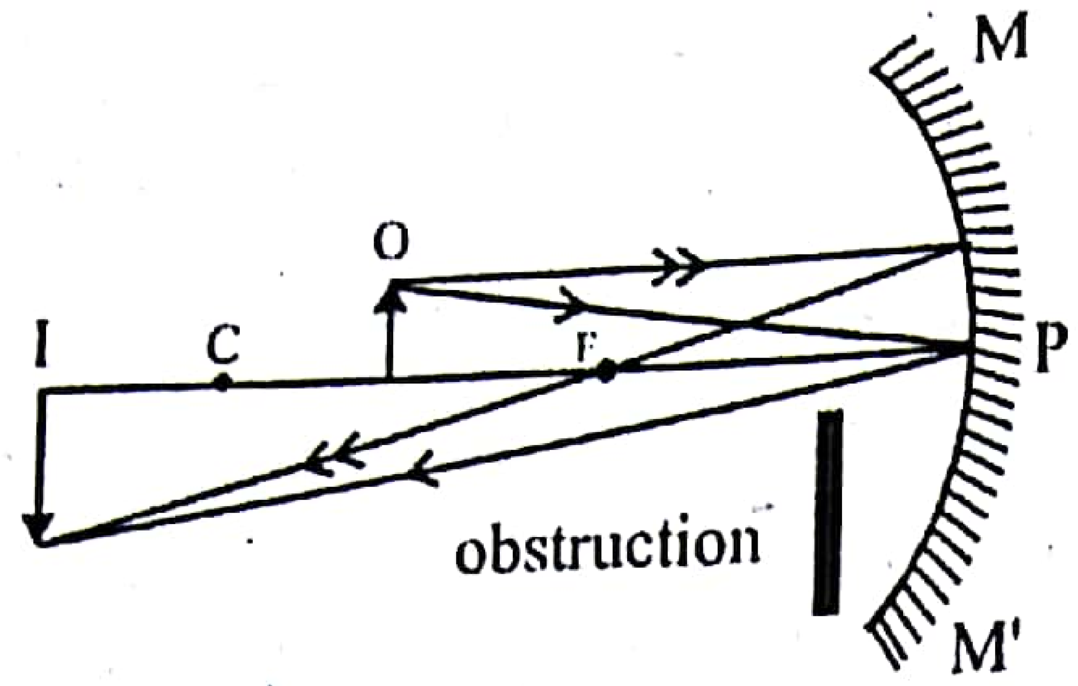
$m = -1$

Larger than object, object is between C & F and image between C and ∞



$m > -1$

3. As every part of a mirror forms a complete image, if a part of the mirror (say half) is obstructed (say covered with black paper), full image will be formed but intensity will be reduced (to half).



4. If an object is moved at constant speed towards a concave mirror from infinity to focus, the image will move slower in the beginning and faster later on, away from the mirror. This is because in the time the object moves from ∞ to C, the image will move from F to C and when the object moves from C to F, the image will move from C to ∞. At C the speed of object and image will be equal.

Note : If the object is moved from F to ∞ at constant speed, the image will move faster in the beginning and slower later on towards the mirror.

5. As focal length of a spherical mirror $f(= R/2)$ depends only on the radius of mirror and is independent of wavelength of light and refractive index of medium so the focal length of a spherical mirror in air or water and for red or blue light is same. This is also why the images formed by mirrors do not show chromatic aberration.

6. In case of spherical mirror if $R \to \infty$ (i.e. it becomes plane), $f = R/2 = \infty$, the mirror formula

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ reduce to $\frac{1}{v}+\frac{1}{u}=0$,

i.e. $v = -u$

i.e., the image lies at same distance behind the mirror as the object is in front of it. This in turn verifies the correctness of mirror formula.

7. In case of spherical mirrors if we plot a graph between :

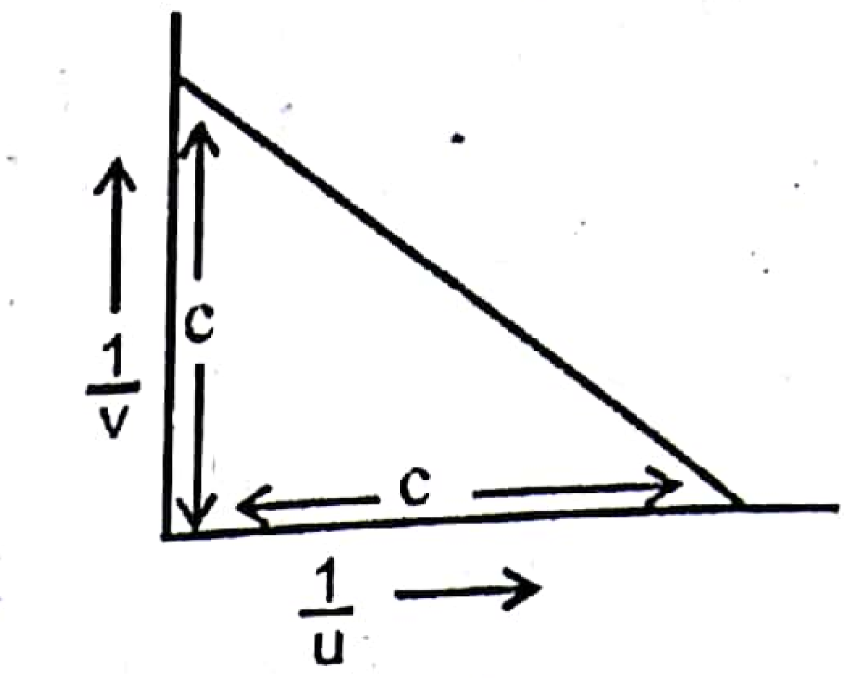
(a) (1/u) and (1/v), the graph will be a straight line with intercept (1/f) with each axis as .

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

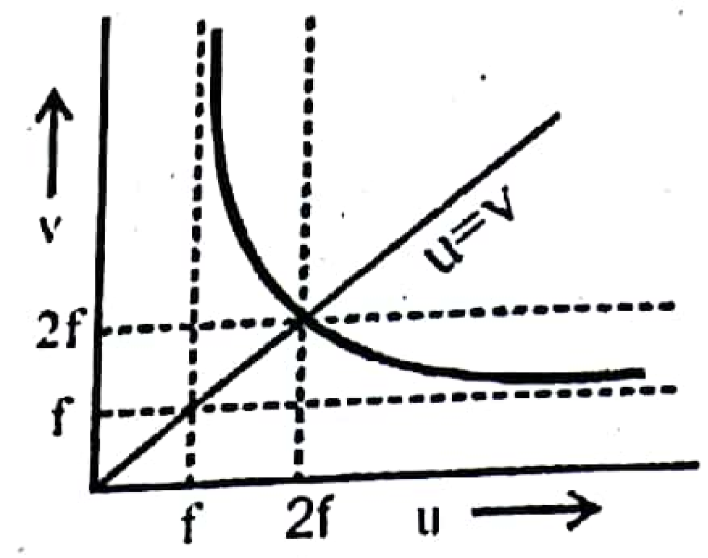
becomes $y + x = c$ with $c=\frac{1}{f}$. This is shown in figure (A).

(b) u and v, the graph will be a hyperbola as for $u = f$, $v = \infty$ and for $u = \infty$, $v = f$.

A line $u = v$ will cut this hyperbola at (2f, 2f). This all is shown in figure (B)

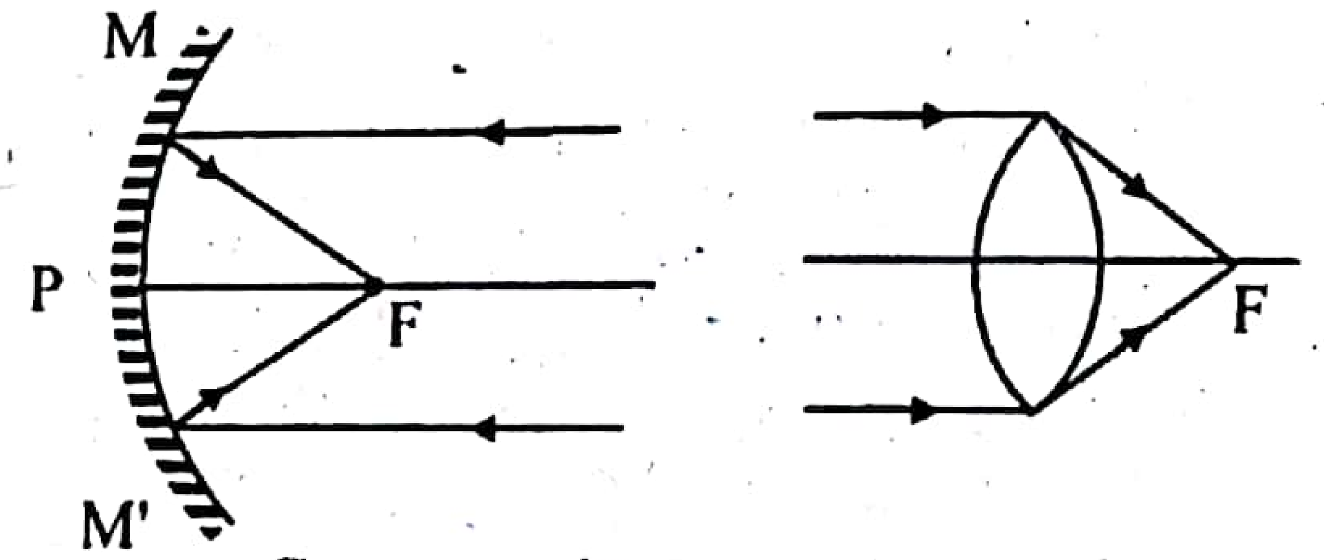


(A)



(B)

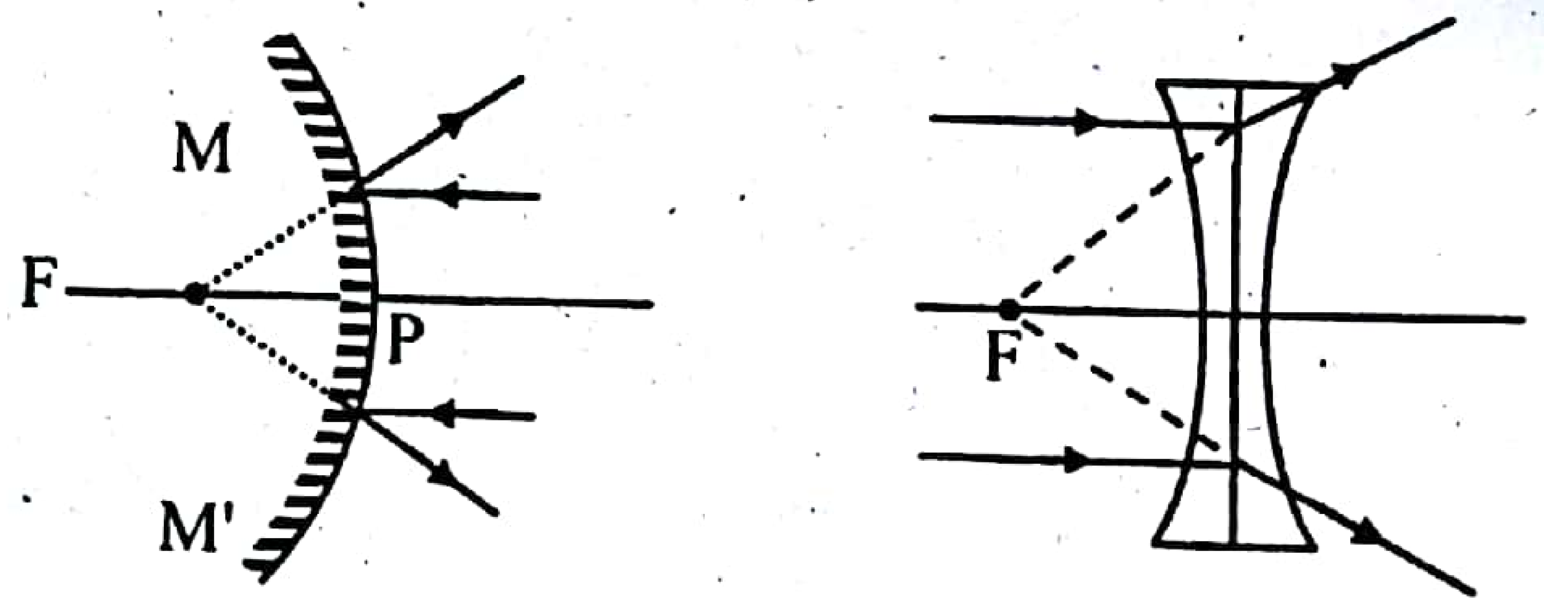
8. Concave mirror behaves as convex lens (both convergent) while convex mirror behaves as concave lens (both divergent). This is shown in figure (C) and (D)



Concave-mirror Convex - lens

Convegent behaviour

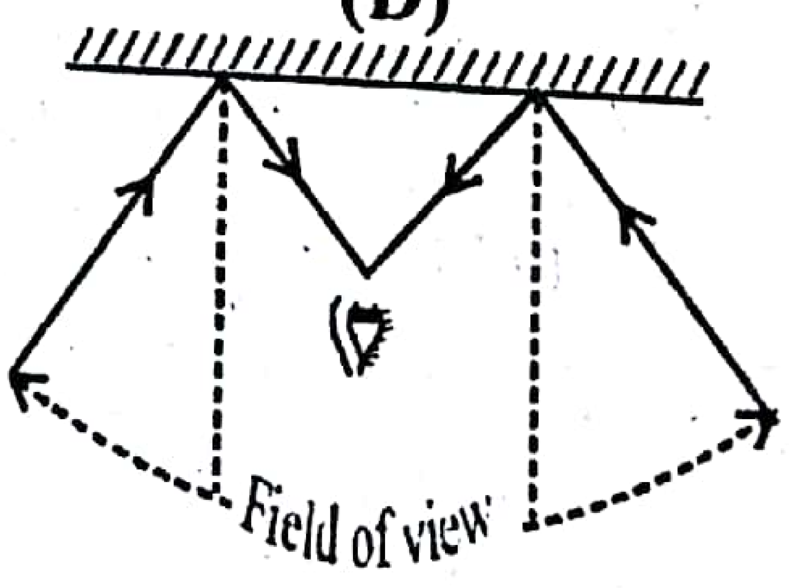
(C)



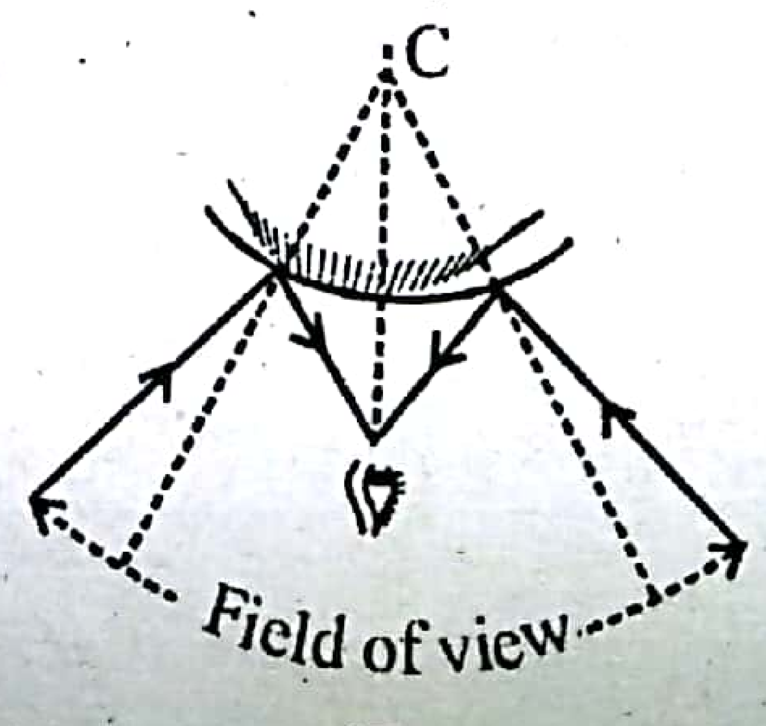
Convex-mirror Concave-lens

Divergent behaviour

(D)



(E)

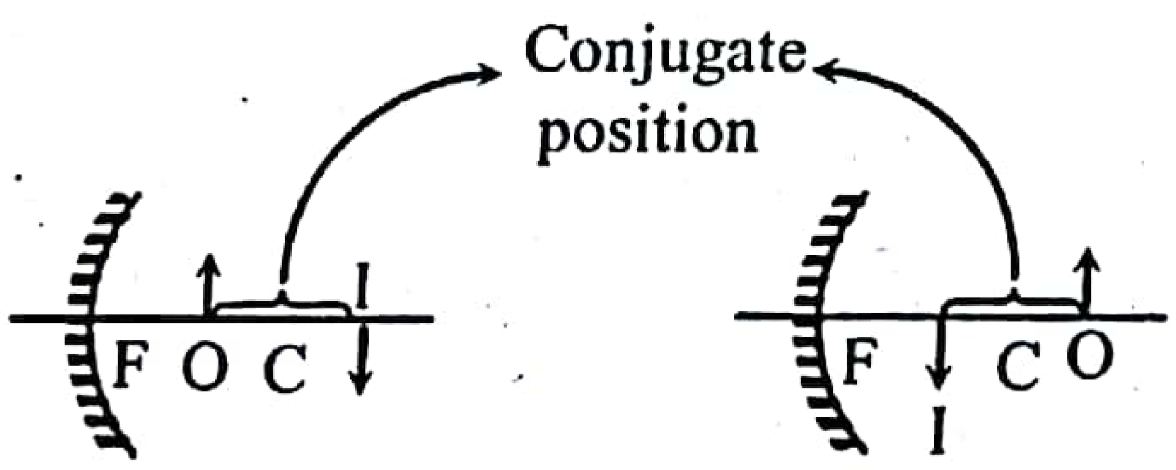


(F)

9. As **convex mirror** gives erect, virtual and diminished image. In convex mirror, the field of view is increased [Figure F] as compared to plane mirror [Figure E]. This is why it is used as rear-view mirror in vehicles. Concave mirrors give enlarged, erect and virtual image (if object is between F and P), so these are used by dentists for examining teeth. Further due to their converging property concave mirrors are also used as reflectors in automobile head lights and search lights and by ENT surgeons in ophthalmoscope.

12. NEWTON'S FORMULA

1. Position of real object and real image are interchangeable.



If I_1 and I_2 is the height of the image

Then, $h_0 = \sqrt{I_1 I_2}$

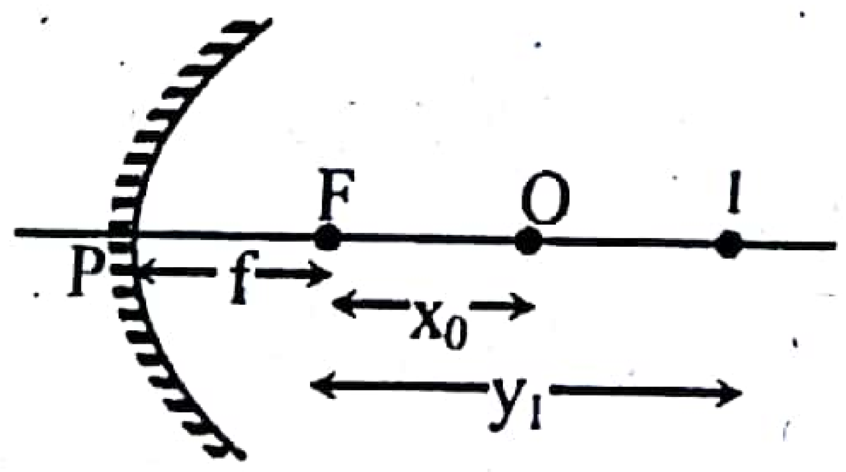
$$\Rightarrow \quad I_1 = -\frac{v}{u}h_0$$

$$I_2 = -\frac{u}{v}h_0$$

2. If distance of object and image is measured from focus instead of pole

then ,$x_o y_i = f^2$

Where ,x_o ,y_i and f are object distance, image distance and focal length respectively.



KEY CONCEPT

1. INTRODUCTION

The phenomenon of change in path of light as it goes from one medium to another is called refraction. Refraction phenomenon can be categories, depending upon the type of separating surface which can be further subdivided depending upon the shape of the medium as follows :

- Refraction
  - At plane surface
    - By Plane
    - By slab
    - By prism
  - At spherical surface
    - By spherical surface
    - By lences
  - At any other surface

Let us study the refraction phenomenon for individual cases :

2. ABSOLUTE REFRACTIVE INDEX

The refractive index (μ) of a medium is defined as the ratio of the speed of light in vacuum (c) to the speed of light in the medium (v),

i.e., $\mu = c/v$

Note:

1. It is a scalar quantity without any unit or dimension.
2. If ε_0 and μ_0 are electric permittivity and magnetic permeability of vacuum while ε and m are that of a given medium then,

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \quad \text{and} \quad v = \frac{1}{\sqrt{\varepsilon\mu}}$$

so that $\mu = c/v = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\varepsilon_r\mu_r}$

3. For vacuum, $\mu = 1$ as $v = c$
4. Absolute refractive index of any medium is greater than one.
5. Refractive index of a medium also depends on the wavelength of light used. As per Cauchy's formula

$\mu = A + B/\lambda^2 + ...$

longer the wavelength smaller is the refractive index.

3. RELATIVE REFRACTIVE INDEX

The relative refractive index of two media is equal to the ratio of their absolute refractive indices.

$$\therefore \quad {}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2}$$

$$= \frac{\text{Velocity of light in first medium}}{\text{Velocity of light in second medium}}$$

Note :

1. $${}_2\mu_1 = \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1}$$

$$\Rightarrow \quad {}_1\mu_2 \times {}_2\mu_1 = \frac{v_1}{v_2} \times \frac{v_2}{v_1} = 1$$

$$\therefore \quad {}_1\mu_2 = \frac{1}{{}_2\mu_1}$$

2. If d_{AC} and d_{AP} are the actual and apparent distances of the object from the plane of the boundary when looked from μ_2 to μ_1 the

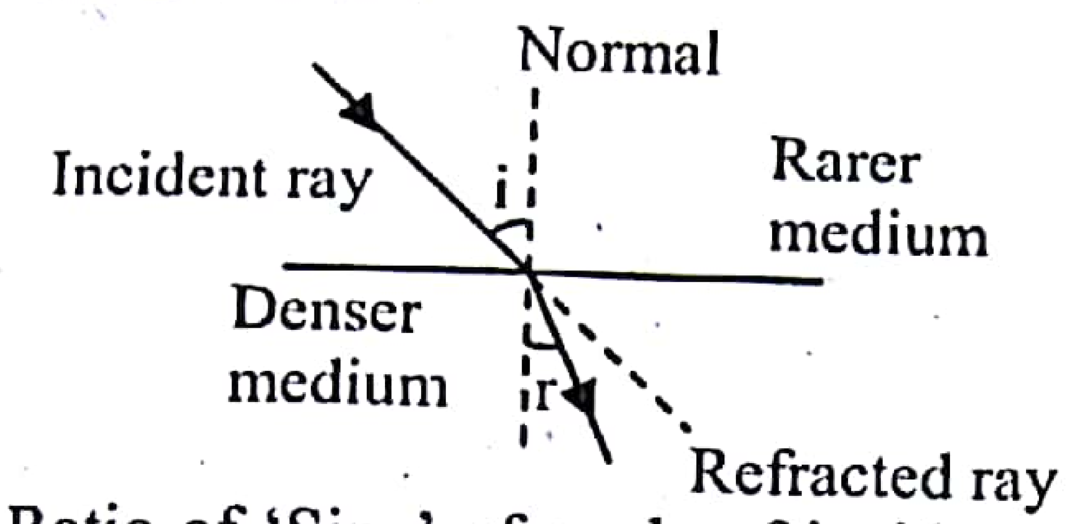
$${}_2\mu_1 = \frac{d_{AC}}{d_{AP}}$$

4. LAWS OF REFRACTION

(a) Frequency (and hence colour) and phase do not change (while wavelength and velocity changes)

(b) Incident ray, refracted ray, and normal always lie in the same plane.

(c) Snell's Law :



Ratio of 'Sine' of angle of incidence to angle of refraction is always a constant, i.e., for all values of i & r –

$$\frac{\sin i}{\sin r} = \text{Constant} = {}_1\mu_2$$

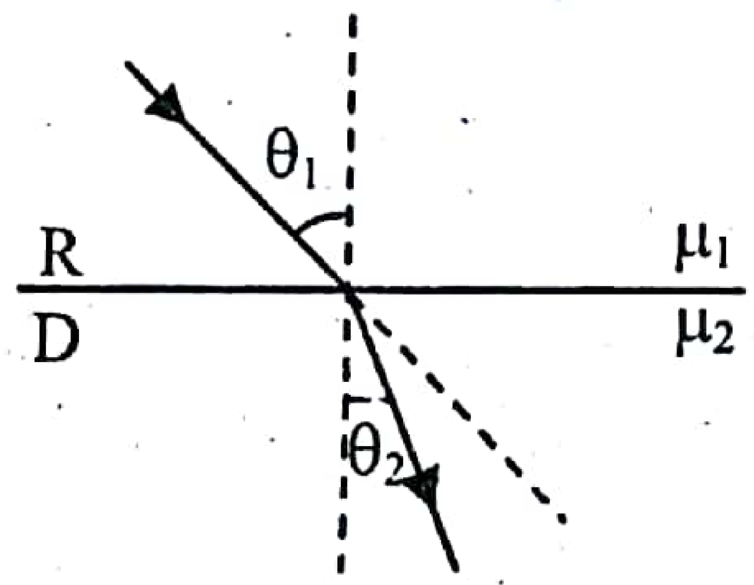
$$= \frac{\text{Velocity of light in first medium}}{\text{Velocity of light in second medium}}$$

$$= \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

5. APPLICATION OF SNELL'S LAW

(A) When light passes from rarer to denser medium it bends toward the normal.

Using Snell's law



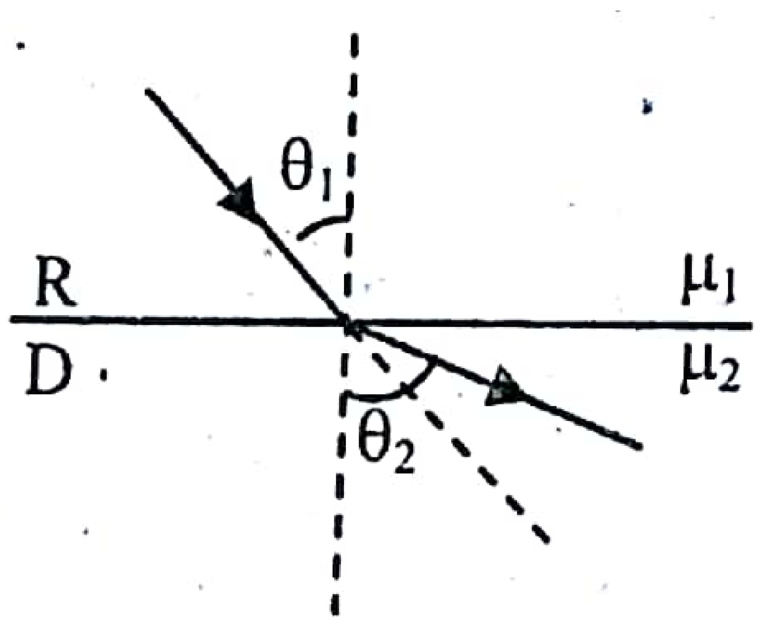
$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\mu_2}{\mu_1}$$

Thus if $\mu_2 > \mu_1$ then $\theta_2 < \theta_1$

(B) When light passes from denser to rarer medium it bends away from the normal.

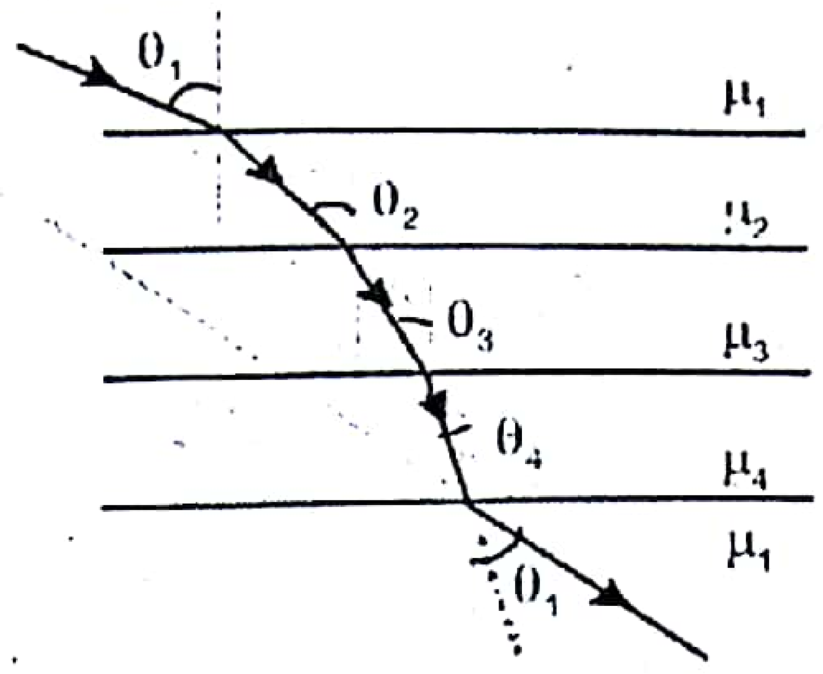
From Snell's law.



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\mu_2}{\mu_1}$$

Thus, if $\mu_2 < \mu_1$. Then $\theta_2 > \theta_1$

(C) When light propagates through a series of layers of different medium, then according to Snell's law



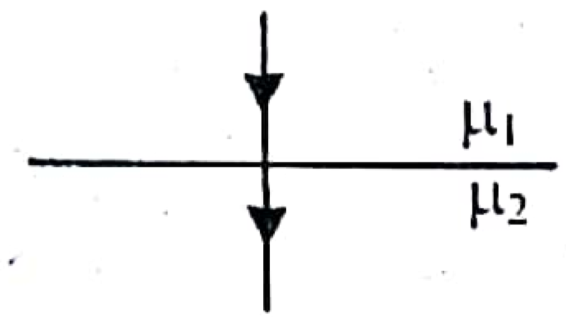
$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 = \mu_3 \sin\theta_3 = ...$

$= \text{constant}$

(D) CONDITIONS OF NO REFRACTION :

(i) If light is incident normally on a boundary i.e., $\angle i = 0°$,

Then from Snell's law,



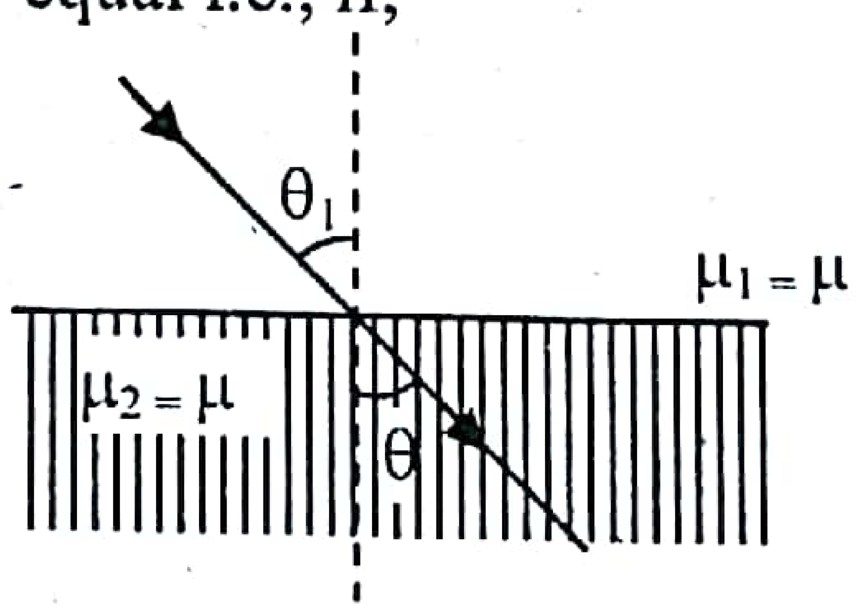
$\mu_1 \sin 0 = \mu_2 \sin r$,

$\Rightarrow \sin r = 0$ i.e. $\angle r = 0$ i.e.,

light passes undeviated from the boundary.

(So boundary will be invisible)

(ii) If the refractive indices of two media are equal i.e., if,



$\mu_1 = \mu_2 = \mu$,

Then from Snell's law

$\mu \sin i = \mu \sin r$

$\Rightarrow \angle i = \angle r$

i.e., ray passes undeviated from the boundary with

$\angle i = \angle r \neq 0$ and boundary will not be visible.

This is also why a transparent solid is invisible in a liquid

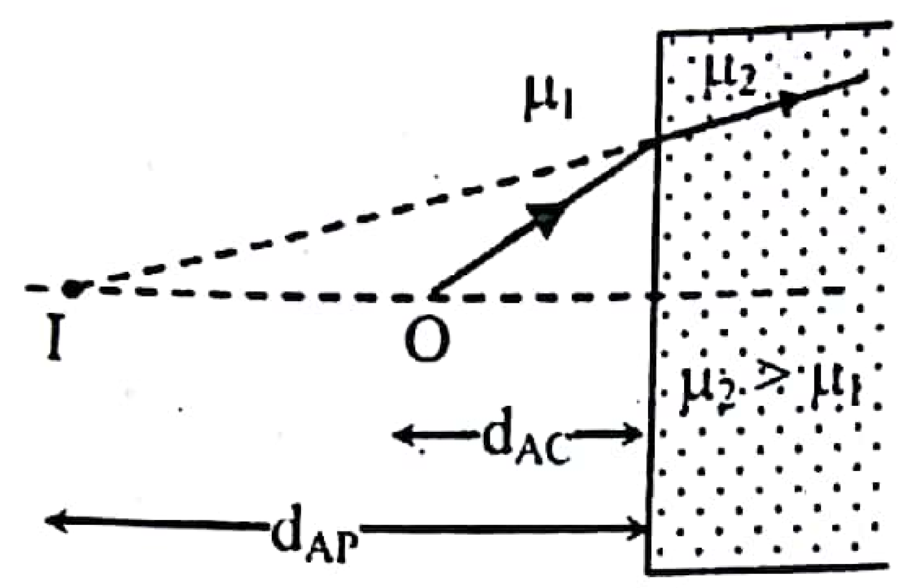
if $\mu_S = \mu_L$

(E) RELATION BETWEEN OBJECT AND IMAGE DISTANCE :

An object O placed in first medium (refractive index μ_1) is viewed from the second medium (refractive index μ_2). Then the image distance d_{AP} and the object distance d_{AC} are related as

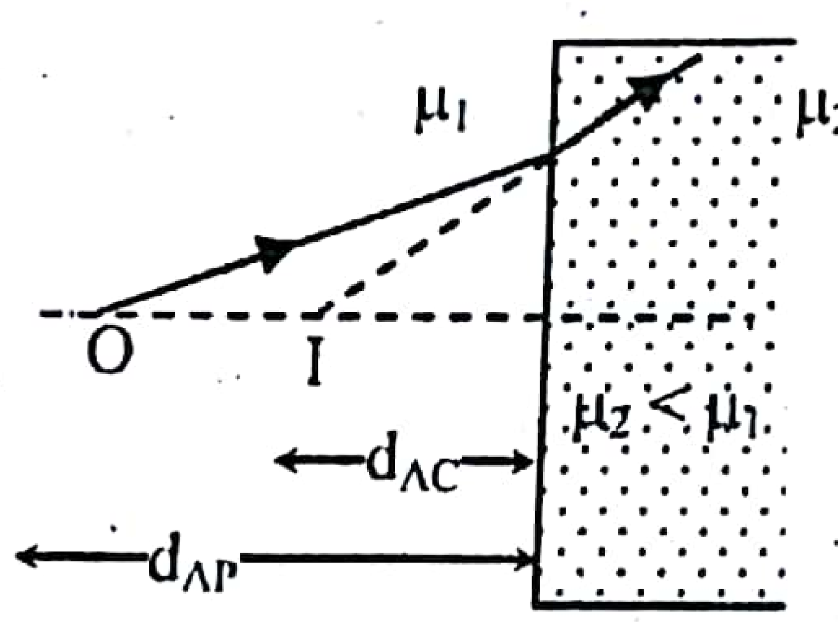
$$d_{AP} = \left(\frac{\mu_2}{\mu_1}\right) d_{AC}$$

(i)



If $\mu_2 > \mu_1$, i.e.,
when the object is observed from a denser medium,
It appears to be farther away from the interface,
i.e., $d_{AP} > d_{AC}$

(ii)

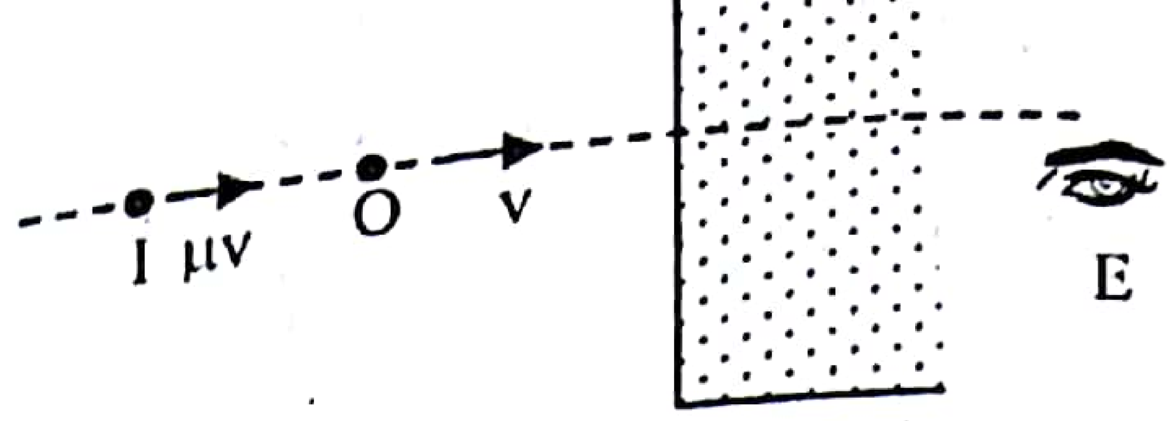


If $\mu_2 < \mu_1$, i.e.,
when the object is observed from a rarer medium.
it appears to be closer to the interface,
i.e., $d_{AP} < d_{AC}$

Note : The above formula is applicable only for normal view or paraxial ray assumption.

(F) RELATION BETWEEN OBJECT AND IMAGE VELOCITIES

(i) If an object O moves toward the plane boundary of a denser medium then the image appears to be farther but moves



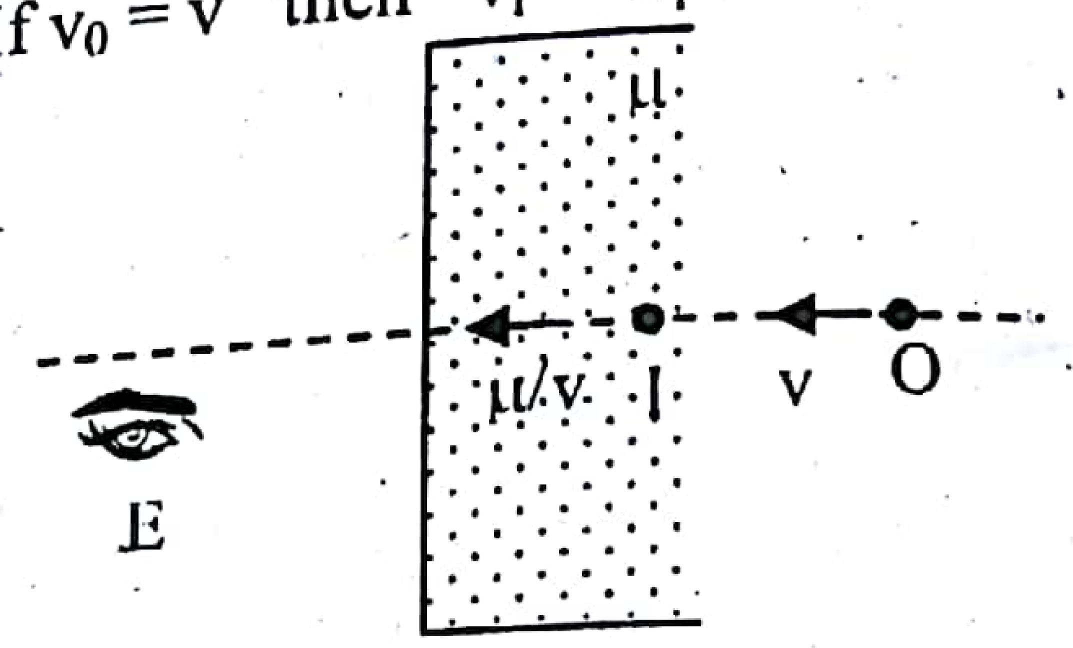
faster to an observer in denser medium. If $v_0 = v$ then

$v_I = \mu v$

Where , v_0 & v_I represents object and image velocities respectively.

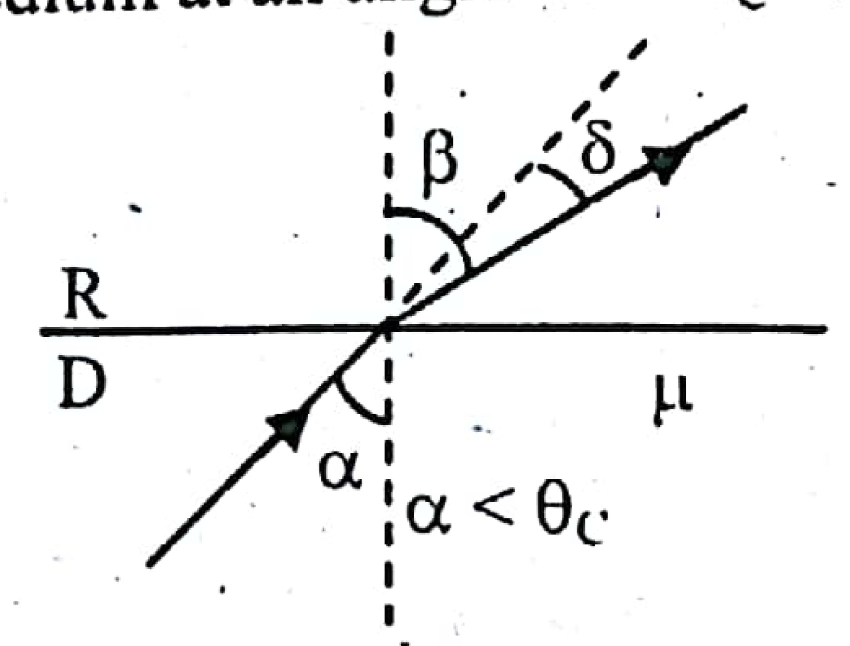
(ii) If an object O moves toward the plane boundary of a rarer medium then the image appears to be closer but moves slower to an observer in rarer medium.

If $v_0 = v$ then $v_I = v/\mu$



(G) Deviation (δ) :

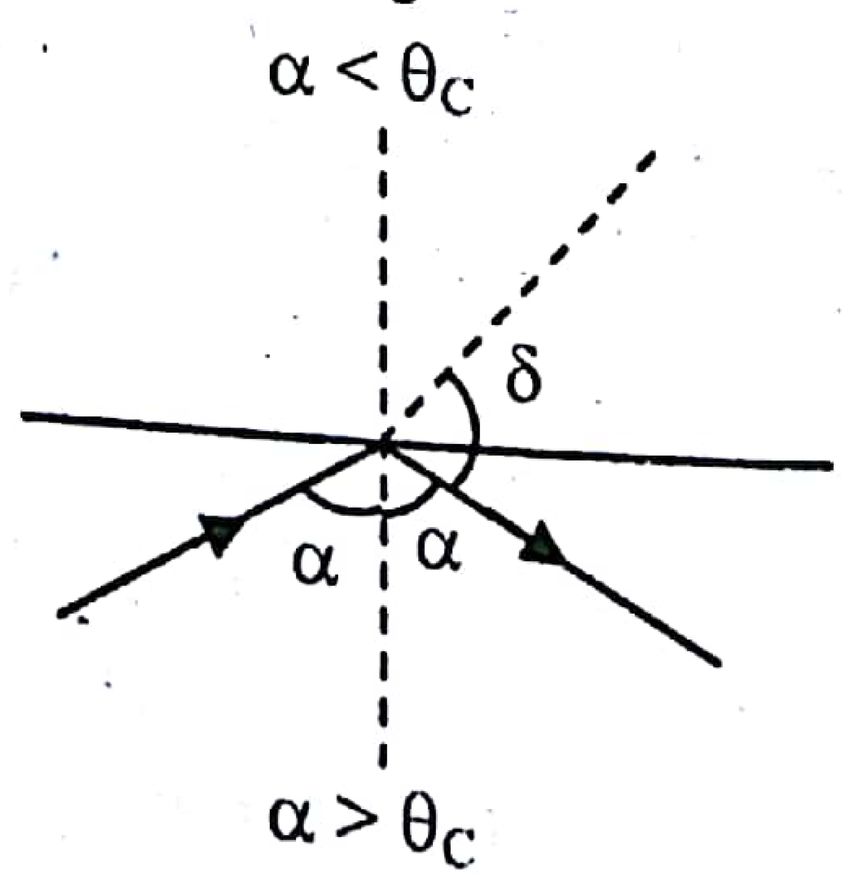
(i) A light ray travelling from a denser to a rarer medium at an angle $\alpha < \theta_C$ then deviation.



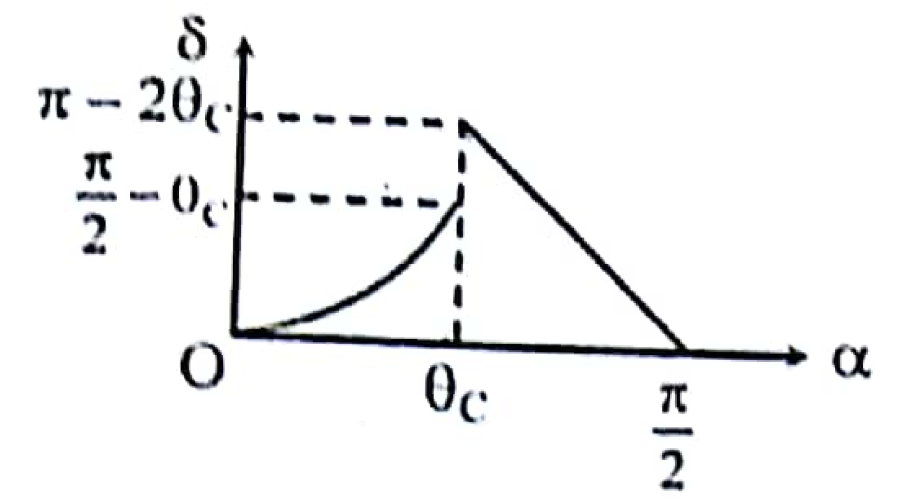
$$\delta = \beta - \alpha = \sin^{-1}(\mu \sin \alpha) - \alpha$$

and $\delta_{max} = \frac{\pi}{2} - \theta_C$

(ii) If light is incident at an angle $\alpha > \theta_C$, Then the angle of deviation is $\delta = \pi - 2\alpha$ and $\delta_{max} = \pi - 2\theta_C$

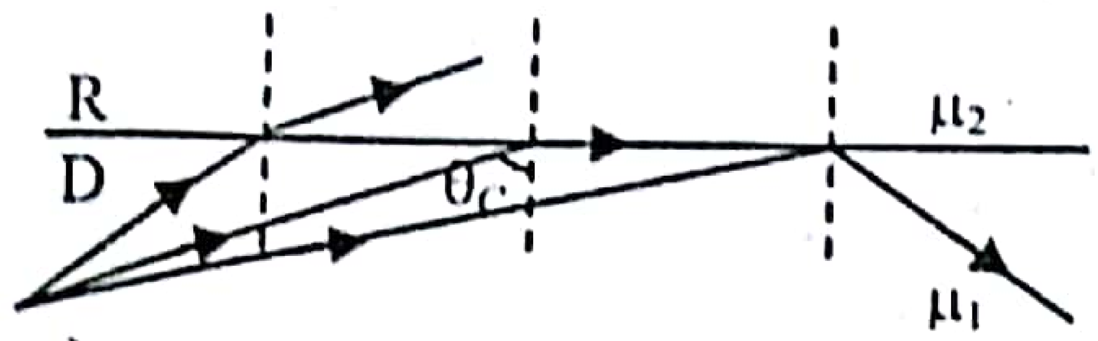


(iii) Graphically the relation between δ & α can be shown as



6. TOTAL INTERNAL REFLECTION

(A) INTRODUCTION :



It is defined as the phenomenon of reflection of light that takes place, when a ray of light travelling in a denser medium gets incident at the interface of the two media, at an angle greater than the critical angle for that pair of media, and reflects back to the denser medium.

At the critical angle (θ_C), the refracted ray just grazes the boundary between two media.

Using Snell's law,

$\mu_1 \sin \theta_C = \mu_2 \sin 90°$

$\Rightarrow \theta_C = \sin^{-1} (\mu_2 / \mu_1)$

(B) CRITICAL ANGLE :

It is the angle of incidence (while going from denser to rarer medium) of light for which angle of refraction is 90°. It is denoted by θ_C

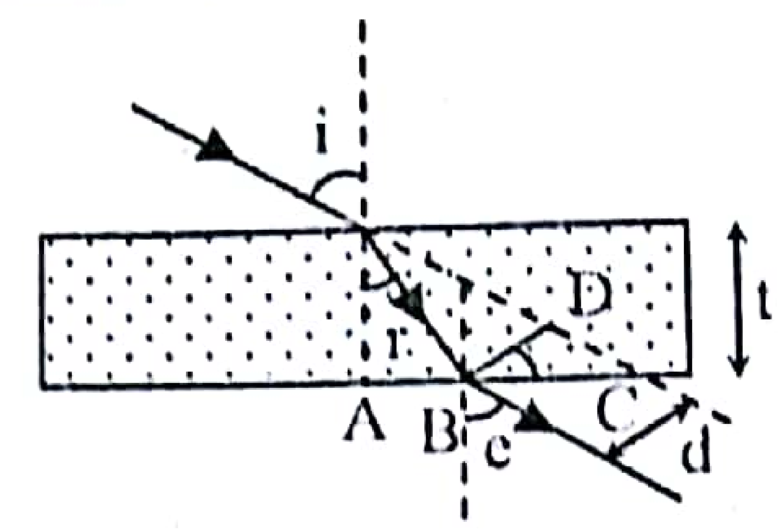
7. REFRACTION BY SLAB

When light falls on the surface of a different medium of thickness 't'and emerges out into the previous medium, Which comes out to be parallel to incident ray at some distance 'd', (known as lateral displacement)

Note : Refracting surfaces are parallel to each other.

7.1 CALCULATION OF LATERAL DISPLACEMENT

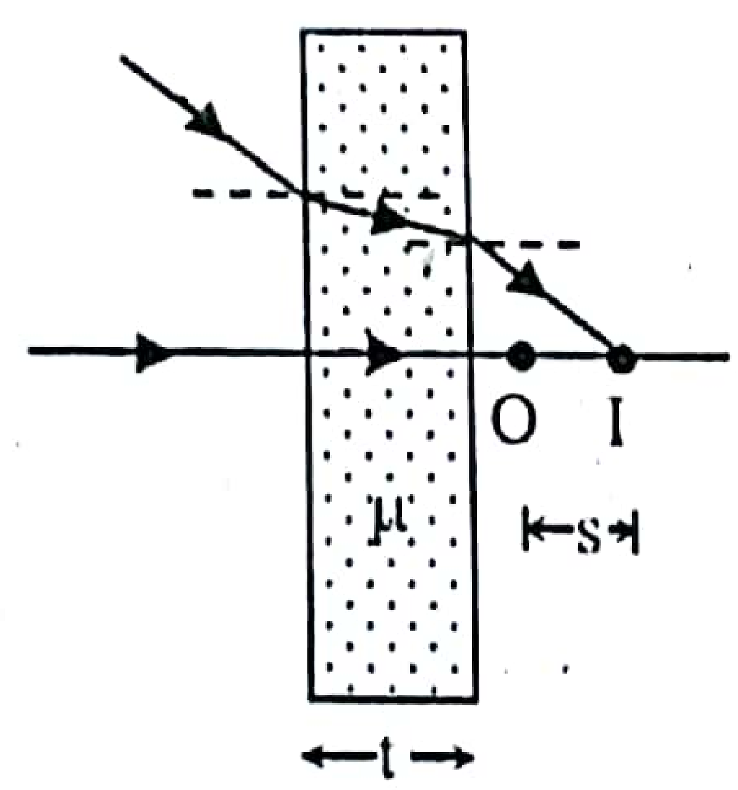
As the refracting surfaces are parallel, incident ray and emergent ray are parallel, i.e., The light ray under goes zero deviation ($\delta = 0$). i.e., $i = e$



The lateral displacement of the ray is the perpendicular distance between the incident and emergent ray $= t\dfrac{\sin(i-r)}{\cos r}$

7.2 APPARENT SHIFT :

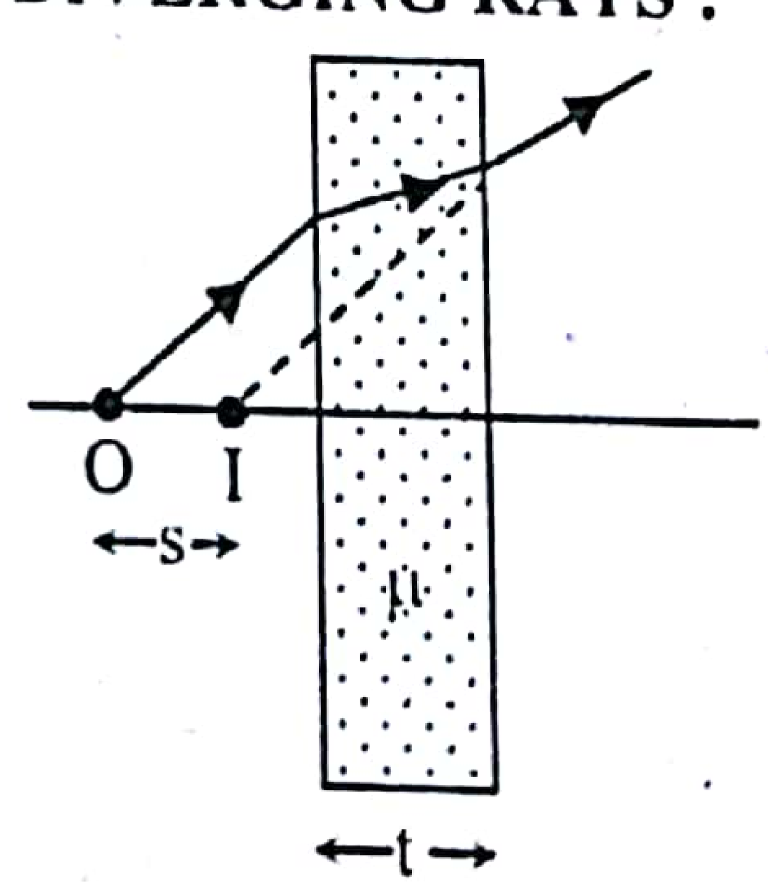
(A) FOR CONVERGING RAYS :



When a slab of thickness t and refractive index μ is placed in the path of a convergent beam, then the point of convergence is shifted by

$$S = \left(1 - \frac{1}{\mu}\right)t$$

(B) FOR DIVERGING RAYS :



When the same slab is placed in the path of divergent beam, then the point of divergence is shifted by,

$$S = \left(1 - \frac{1}{\mu}\right)t$$

Note :

1. The shift 'S' is always in the direction of light.
2. If the slab is made of air and surrounding medium is of refractive index μ, Then the apparent shift would be $S = t(\mu - 1)$
3. If n number of slabs with different thickness and refractive index are placed between the observer and the object, then the total apparent shift is equal to the summation of the individual shifts.

$\therefore \quad S = S_1 + S_2 + ... + S_n$

$$= t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right) + ... + t_n\left(1 - \frac{1}{\mu_n}\right)$$

$$= \sum_{i=1}^{n} t_i\left(1 - \frac{1}{\mu_i}\right)$$

4. If there are n number of slabs with different thickness and refractive index, one over the other then

$$d_{AC} = t_1 + t_2 + ... + t_n$$

And $d_{AP} = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + ... + \frac{t_n}{\mu_n}$

So, $\mu = \frac{d_{AC}}{d_{AP}}$

$$= \frac{t_1 + t_2 + ... + t_n}{\left(\frac{t_1}{\mu_1}\right) + \left(\frac{t_2}{\mu_2}\right) + ... + \left(\frac{t_n}{\mu_n}\right)} = \frac{\Sigma t_i}{\Sigma\left(\frac{t_i}{\mu_i}\right)}$$

In case of two liquids with $t_1 = t_2$

$\mu = \frac{2\mu_1\mu_2}{\mu_1 + \mu_2}$ i.e., harmonic mean.

8. PRISM

8.1 DEFINITION :

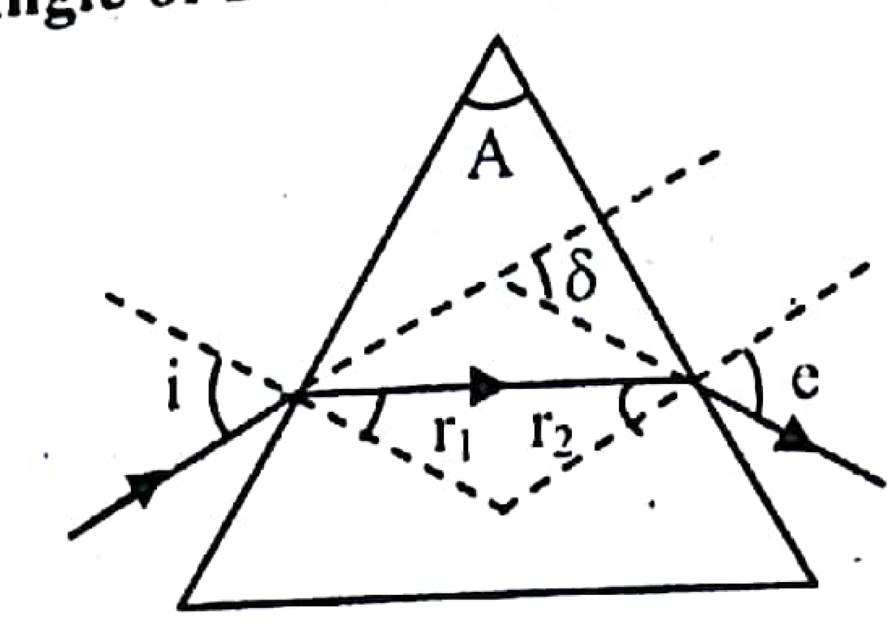
It is a transparent medium whose refracting surfaces are not parallel but are inclined to each other.

8.2 TERMS RELATED TO PRISM :

(A) Angle of Prism or Refracting Angle (A) :

It is the angle between the faces on which light is incident and from which it emerges.

(B) Angle of Deviation (δ) :

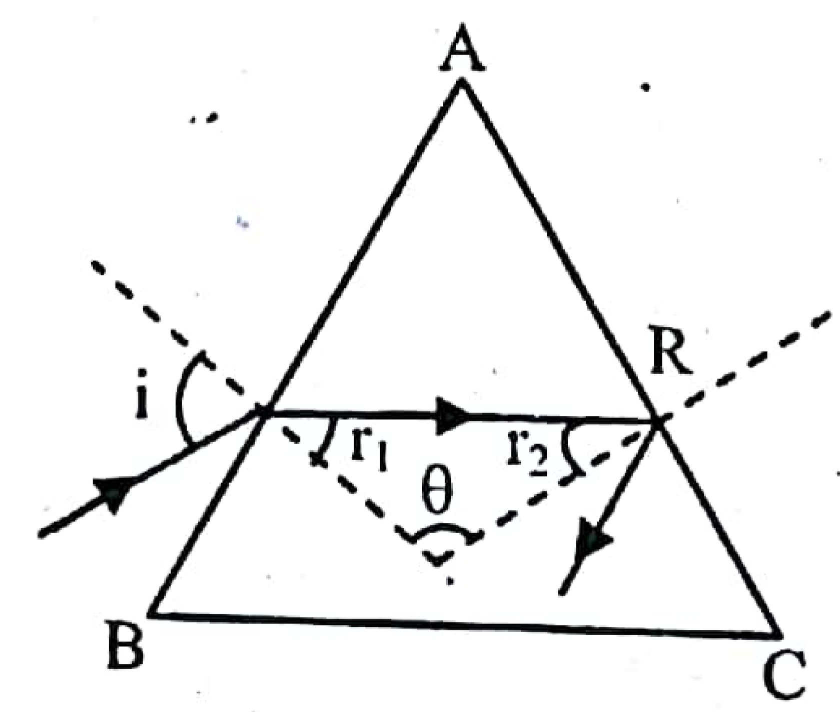


It is the angle between the emergent and the incident ray. In other words, it is the angle through which incident ray turns while passing through a prism.

$\delta = (i - r_1) + (e - r_2) = i + e - (r_1 + r_2)$

$= i + e - A$

8.3 CONDITION OF NO EMERGENCE :



The light will not emerge out of a prism for all values of angle of incidence if at face AB for $i = max = 90°$, at face AC $r_2 > \theta_c$, hence

$A > 2\theta_c$ or $\mu > \text{cosec}(A/2)$

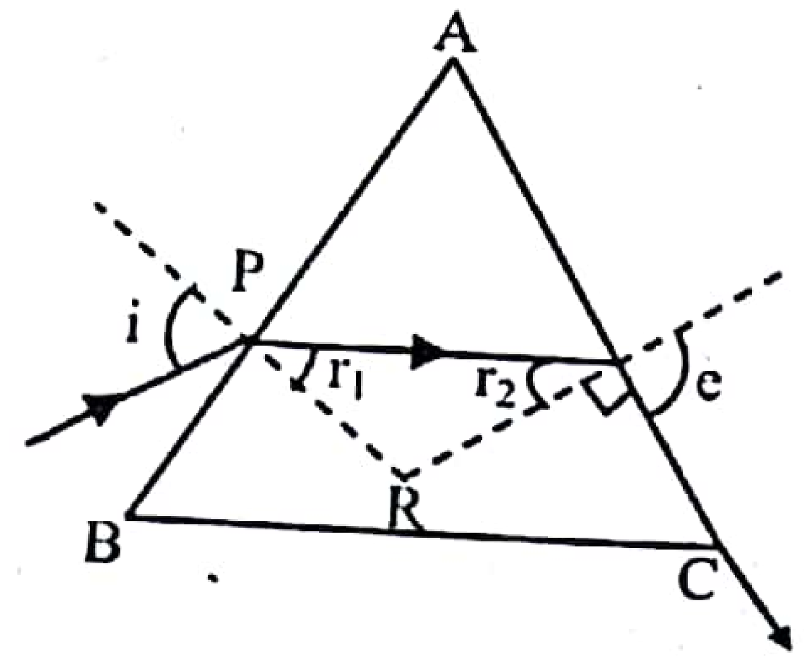
i.e., A ray of light will not emerge out of a prism (what ever be the angle of incidence) if

$A > 2\theta_C$ i.e., if $\mu > \text{cosec}(A/2)$

8.4 CRITICAL ANGLE :

It is the angle of prism above which incident light ray on first surface will not emerge out from the second surface for all possible values of angle of incidence $A = 2\theta_C$

8.5 CONDITION OF GRAZING EMERGENCE :



If a ray can emerge out of a prism , the value of angle of incidence i for which angle of emergence e = 90° is called condition of grazing emergence.

i.e., $r_2 = \theta_c$

$\therefore \quad i = \sin^{-1}\left[\sqrt{\mu^2 - 1}\sin A - \cos A\right]$

Note :

The light will emerge out of a given prism only if the angle of incidence is greater than the condition of grazing emergence.

8.6 CONDITION OF MAXIMUM DEVIATION :

Deviation will be maximum when

$i_{max} = 90°$

So, $\delta_{max} = i_{max} + e - A = 90° + e - A$

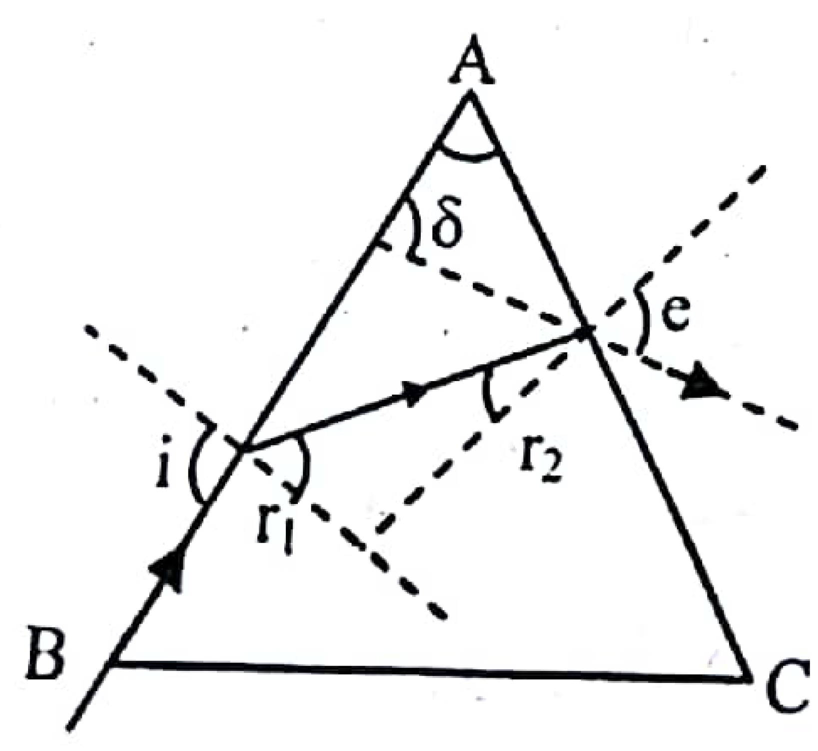
However, when i = 90°,

$\therefore \quad r_1 = \theta_C$

$r_2 = A - \theta_C$

so $e = \sin^{-1}$ [$\mu \sin (A - \theta_C)$]

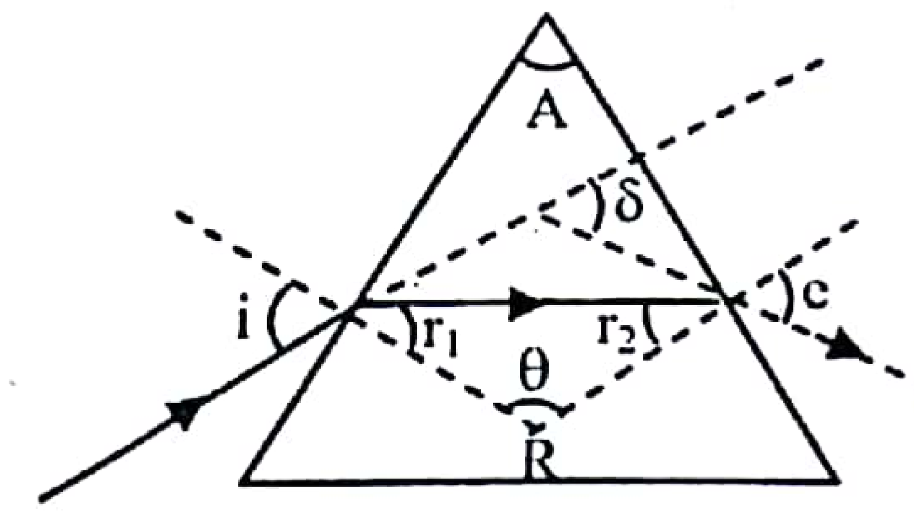
Thus, $\delta_{max} = 90° + \sin^{-1}[\mu \sin (A - \theta_C)] - A$



Note :

This situation is reverse of grazing emergence and may also be viewed as deviation at grazing incidence .

8.7 CONDITION FOR MINIMUM DEVIATION



The minimum deviation occurs when the angle of incidence is equal to the angle of emergence i.e., i = e

$\therefore \quad \boxed{\delta_{min} = 2i - A}$ [As $\delta = i + e - A$]

$$i = \frac{\delta_{min} + A}{2}$$

$\Rightarrow \quad r_1 = r_2 = r$ (say)

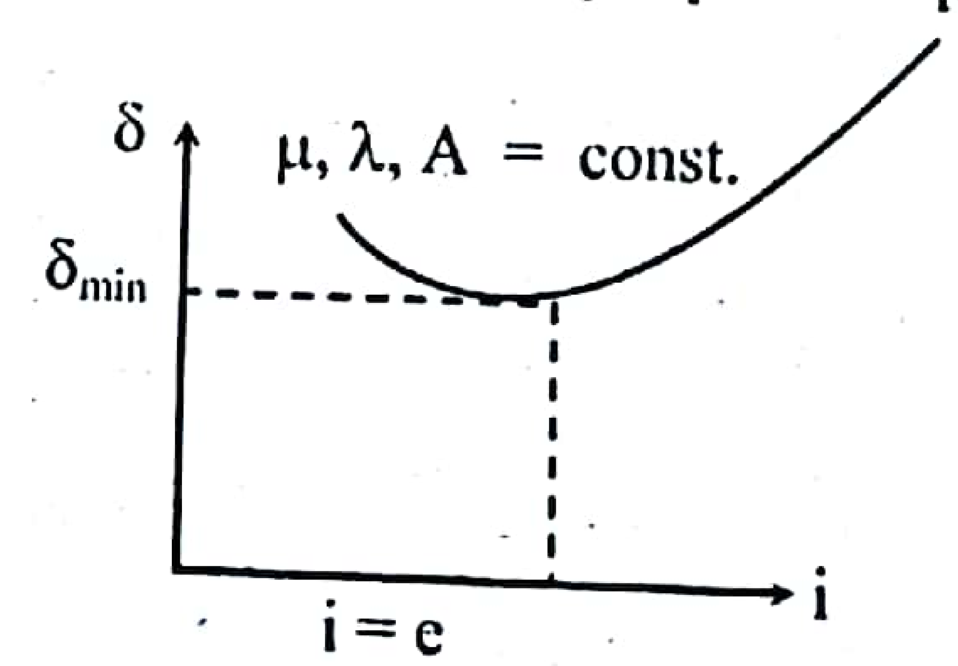
$r = A/2$

$$\boxed{\mu = \frac{\sin i}{\sin r} = \frac{\sin(\delta_{min} + A)}{\sin(A/2)}}$$

Note: In the condition of minimum deviation the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.

8.8 GRAPHICAL REPRESENTATION OF ANGLE OF DEVIATION :

The deviation produced by a prism depends on



(A) Angle of incidence , i

(B) Angle of prism , A

(C) Refractive index of material , μ and

(D) Wavelength of light , λ

(i) δ is first decreasing and then increasing for i < e and i > e respectively.

(ii) $\delta \propto A$

(iii) $\delta \propto (\mu - 1)$

(iv) $\delta \propto \frac{1}{\lambda}$

8.9 THIN PRISMS :

In thin prism the distance between the refracting surfaces is negligible and the angle of prism (A) is very small.

so $\boxed{\delta = A(\mu - 1)}$

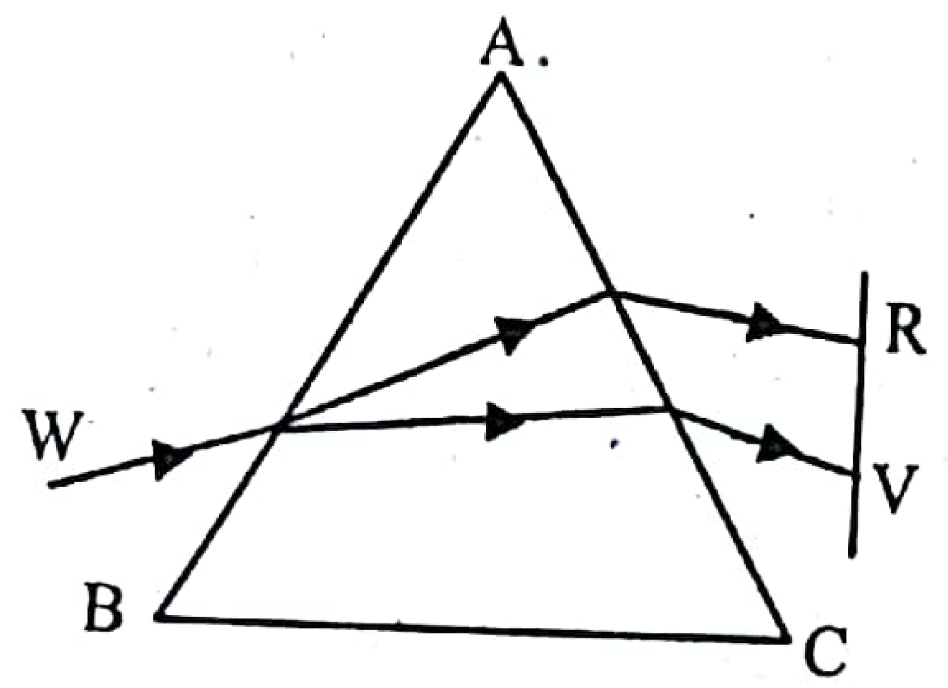
Note :

The deviation for a small angled prism is independent of the angle of incidence.

9. DISPERSION

9.1 PHENOMENON :

It is the phenomenon of splitting of a beam of white light into its constituent colours on passing through a prism.



9.2 CAUSE :

The refractive index of a material depends on the wavelength of light, given by

CAUCHY'S FORMULA

$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ...$ where A, B, C are constants

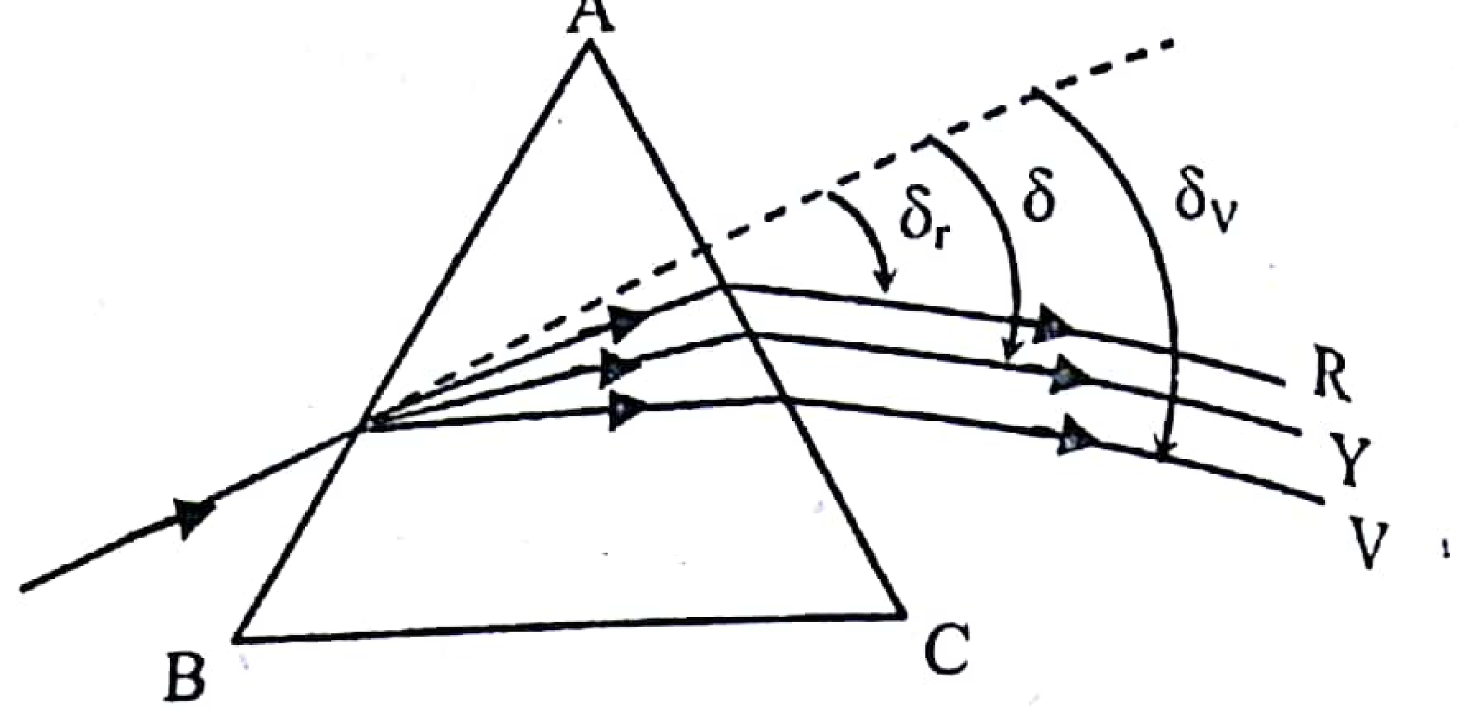
Note :

1. As μ decreases with increase in λ, for visible light μ is maximum for violet and minimum for red.
2. Because of the different refractive indices, light of different colours bend through different angles on refraction .

9.3 ANGULAR DISPERSION

Angular dispersion produced by a prism for white light is the difference in the angles of deviation of two extreme colours i.e., violet and red colours

$$\boxed{(\delta_v - \delta_r) = (\mu_v - \mu_r)A}$$



9.4 DISPERSIVE POWER :

The dispersive power of a prism is defined as the ratio of angular dispersion to the mean deviation produced by the prism. It is represented by ω .

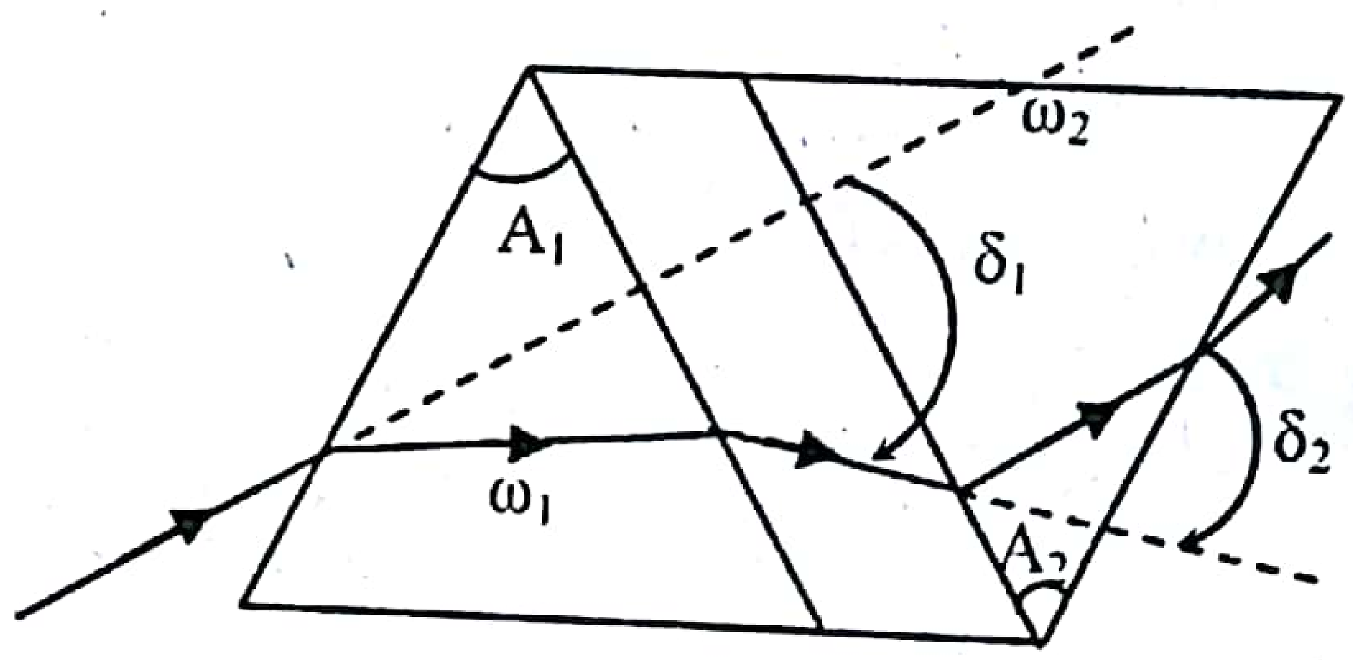
i.e., $\omega = \frac{\Delta\mu}{\mu - 1} = \frac{(\mu_v - \mu_r)A}{(\mu - 1)A}$

Note:

Dispersive power of a prism depends only on nature of material of the prism. However, angular dispersion and mean deviation, both depend also on the angle of prism.

9.5 COMBINATION OF PRISMS :

Two prisms of refracting angles A_1 and A_2 and dispersive power ω_1 and ω_2 are placed symmetrically (as shown in the figure) for a particular ray refractive indices of the two prisms are μ_1 and μ_2 respectively.



Thus deviation & dispersion produced by two prisms are,

$\delta_1 = (\mu_1 - 1) A_1$ and $\delta_2 = (\mu_2 - 1) A_2$

$\theta_1 = (\mu_{v_1} - \mu_{r_1})A_1$ and

$\theta_2 = (\mu_{v_2} - \mu_{r_2})A_2$

(A) DISPERSION WITHOUT DEVIATION :

$\delta = \delta_1 + \delta_2 = 0$

net dispersion = $\delta_1 (\omega_1 - \omega_2)$

(B) DEVIATION WITHOUT DISPERSION :

$\theta = \theta_1 + \theta_2 = 0$

net deviation = $\delta_1\left(1 - \frac{\omega_1}{\omega_2}\right)$

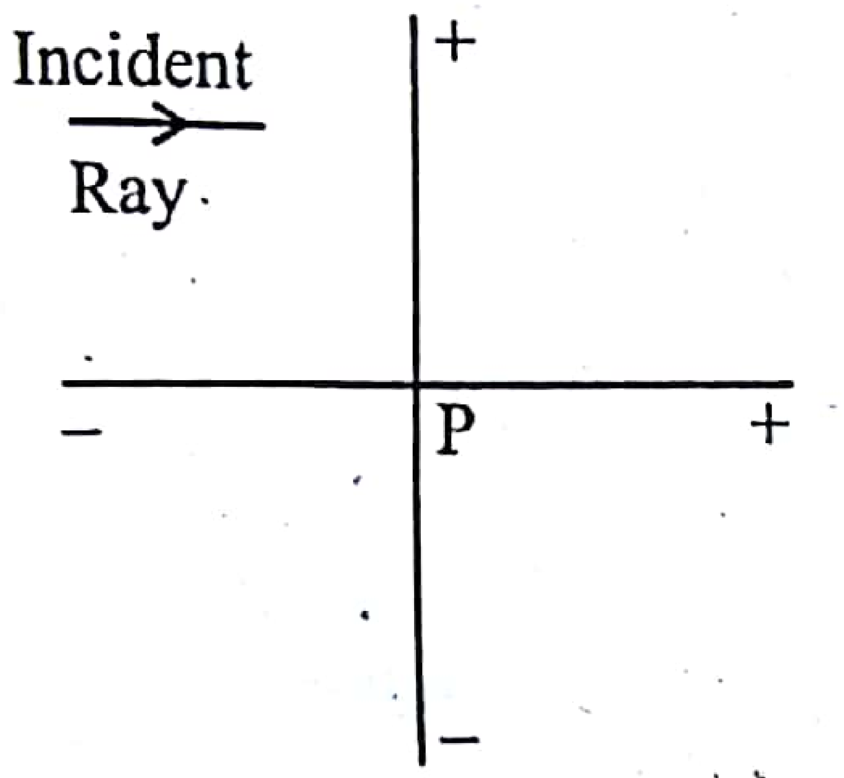
KEY CONCEPT

1. LAW OF REFRACTION AT SPHERICAL SURFACE

When light passes from a medium of refractive index μ_1 to a medium of refractive index μ_2 by a spherical surface of radius of curvature R then the relation between object distance u and image distance v is given by

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

1.1 Cartesian sign convention



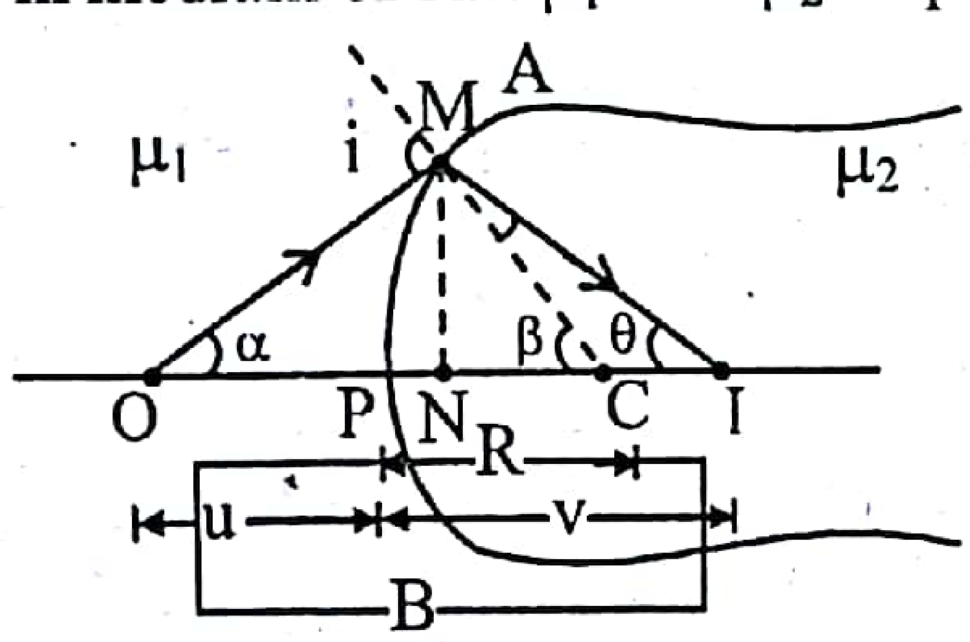
(1) All distances are measured from the pole (P).

(2) Distances measured in the direction of incident rays are taken as positive.

(3) Distances above the principal axis are taken as positive.

(4) Angles measured from the normal in the anticlockwise sense are positive.

Suppose AB is the spherical surface which separates the two medium of refractive index μ_1 and μ_1 .O and I is the position of object and image in medium of R.I. μ_1 and μ_2 respectively.



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

[For small aperture P & N are the same point]

1.2 Terms related to refraction at spherical surfaces

(A) centre of curvature (C)

It is the centre of sphere of which the surface is a part.

(B) Radius of curvature (R)

It is the radius of the sphere of which the surface is a part.

(C) Pole (P)

It is the geometrical centre of the spherical refracting surface .

(D) Principal Axis

It is the straight line joining the centre of curvature to the pole.

(E) Focus (F)

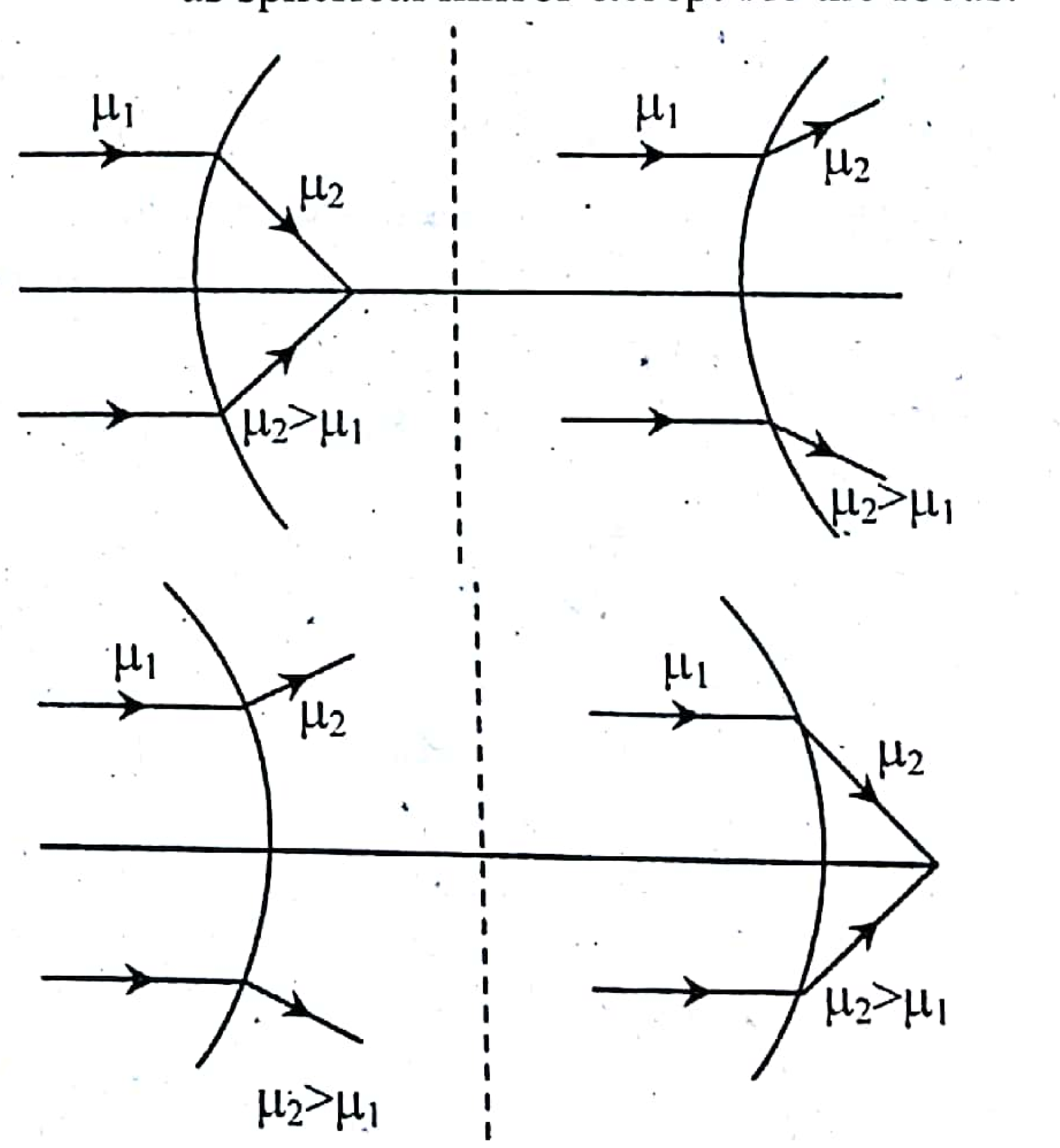
When a narrow beam of rays of light, parallel to the principal axis and close to it (known as paraxial rays) is incident on a refracting spherical boundary , the refracted beam is found to converge or appear to diverge [depending upon the nature of the boundary (concave/canvex) and the refractive index of two media] from a point on the principal axis .This point is called focus.

Note :

(1) It is not always necessary that for convex boundary the parallel rays always converge. Similarly for concave boundary the incident parallel ray may converge or diverge depending upon the refractive index of two media,

(2) Laws of refraction are valid for spherical surface .

(3) Pole, centre of curvature, Radius of curvature, Principal axis etc. are defined as spherical mirror except for the focus.



2. LATERAL MAGNIFICATION

It is defined as

$$m = \frac{h_i}{h_o} = \frac{\text{height of image}}{\text{height of object}} = \frac{\mu_1}{\mu_2} \cdot \frac{v}{u}$$

3. LENS THEORY

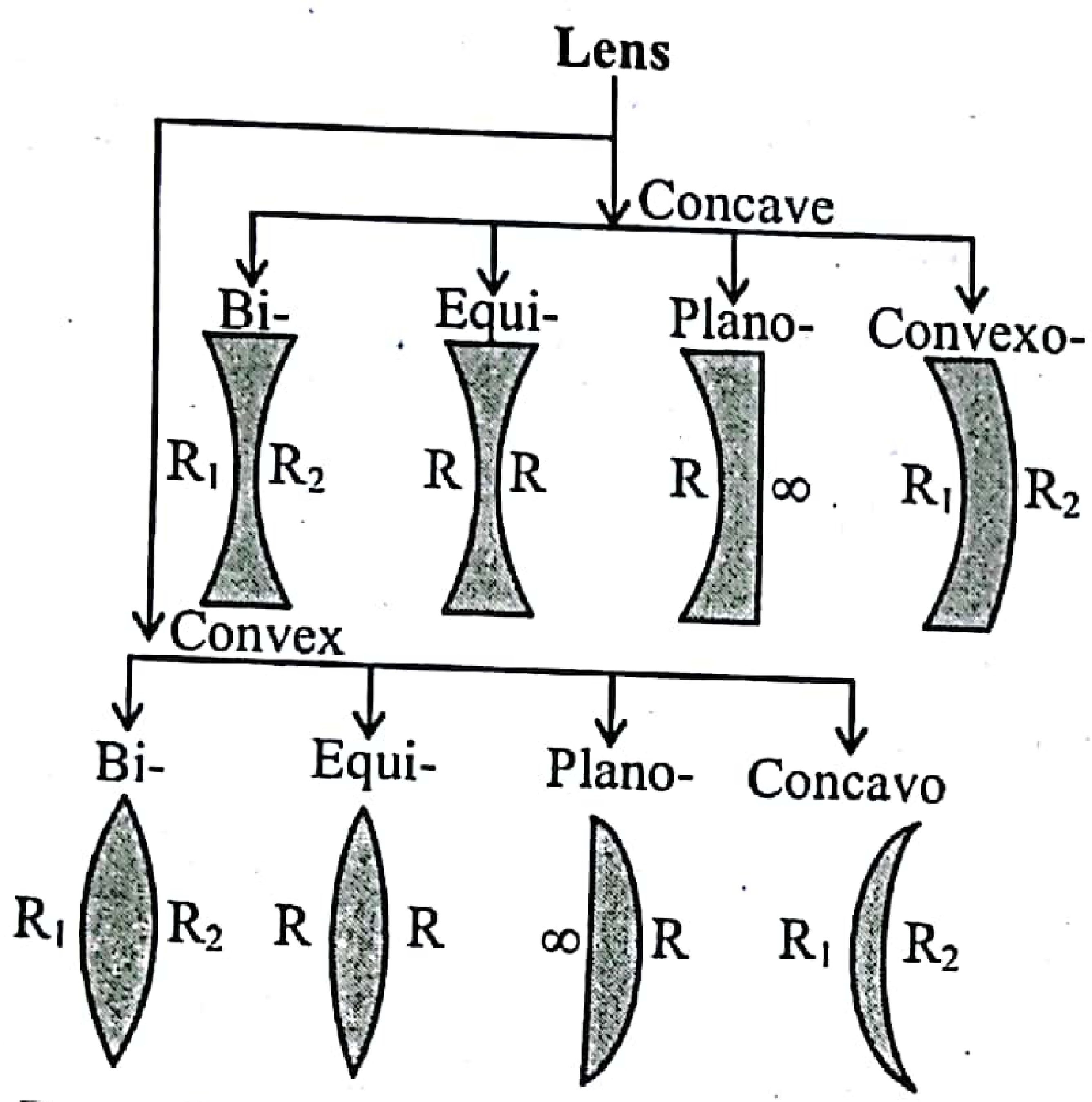
3.1 Definition :

A lens is a piece of transparent material with two refracting surfaces such that at least one is curved and refractive index of its material is different from that of the surroundings.

Note :

Here we will discuss only thin spherical lenses.

3.2 Types of lenses :



Depending upon the shape of the refracting surfaces following types of lenses can be formed:

3.3 Terms related to thin spherical lenses :

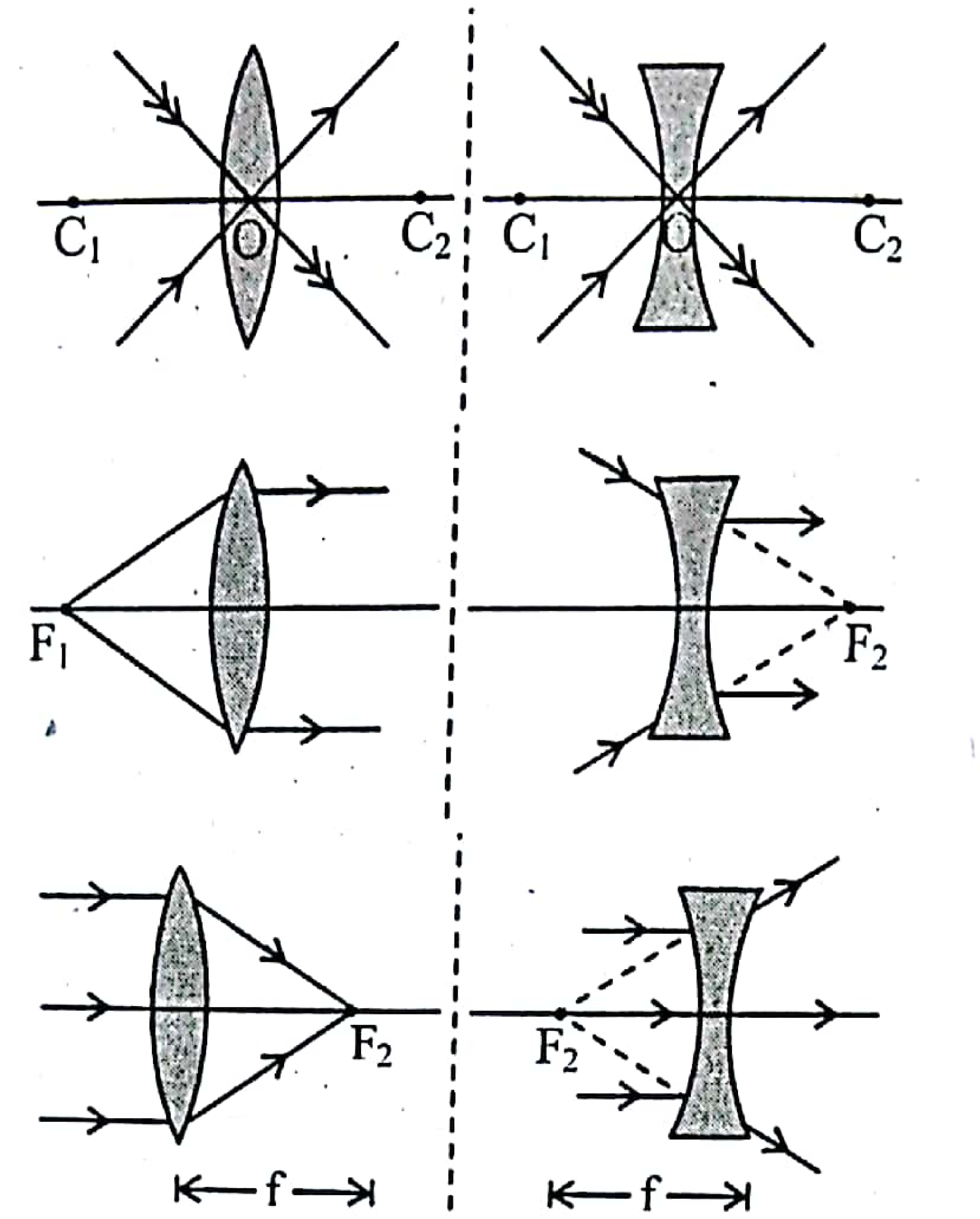
(A) Optical centre (O) is a point for a given lens through which any ray passes undeviated.

(B) Principal Axis (C_1C_2) is a line passing through optical centre and perpendicular to the lens. The centre of curvature of curved surfaces always lies on the principal axis .

(C) Principal Focus :

A lens has two surfaces causing two focul points

(i) **First focal point** is an object point on the principal axis whose image is formed at infinity.



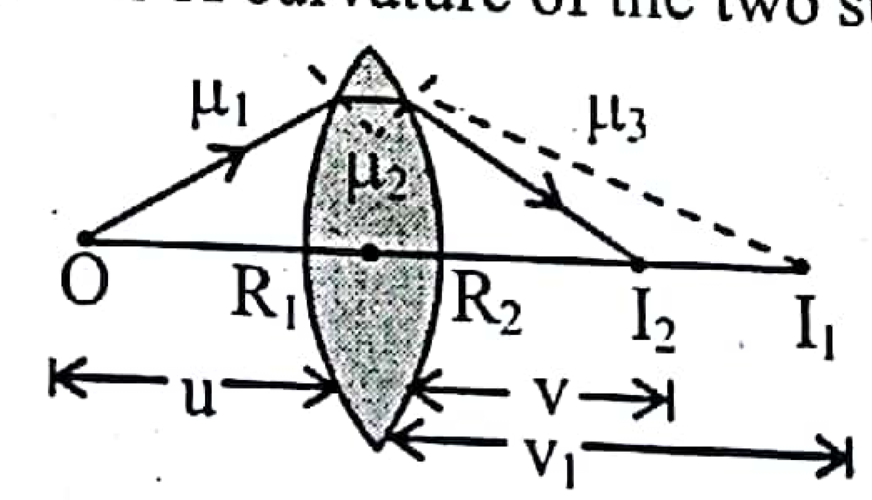
(ii) **Second focal point** is an image point on the principal axis whose object lies at infinity.

(D) Focal length (f) is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appear to converge.

(E) Aperture : In reference to a lens, aperture means the effective diameter of its light transmitting area. So the brightness,i.e., intensity of image formed by a lens which depends on the light passing through the lens will depend on the square of aperture, i.e., $I \propto (\text{aperture})^2$

3.4 Lens -Maker's formula :

It relates the focal length of the lens to the relative refractive index μ of the lens material and the radii of curvature of the two surfaces



$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Where, $\mu = \frac{\mu_2}{\mu_1} = \frac{\text{refractive index of lens}}{\text{refractive index of surrounding}}$

R_1 is the radius of curvature of first surface and R_2 is the radius of curvature of the second surface.

Note :

(1) The **Lensmaker's formula** is applicable for thin lenses only. The value of R_1 and R_2 are to be put in accordance with the cartesian sign convention.

(2) Position of object and image are interchangable . These positions are called – **conjugate position.**

3.5 Lens formula :

The object and image distances of a lens are related to each other as :

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad \text{Where,}$$

$$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

3.6 Lateral magnification :

$$m=\frac{h_i}{h_o}=\frac{v}{u}$$

Note :

(1) If converging ray's fall, the focus is on the other side of the direction of incidence and for diverging ray's focus is on the same region of the direction of incidence.

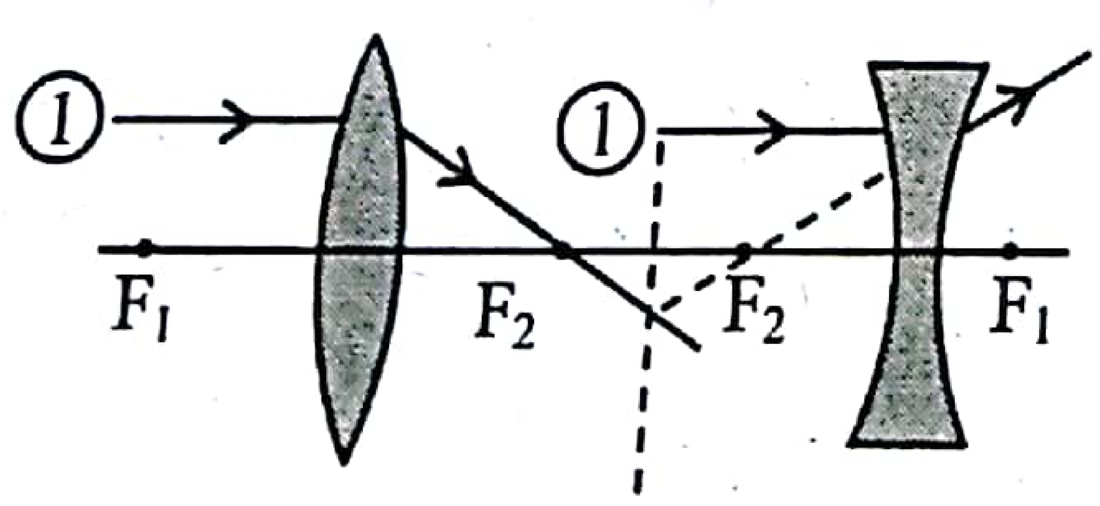
(2) m has negative (positive) value for real- real (real virtual) pair.

(3) Use cartesion sign convention with pole of lens as origin.

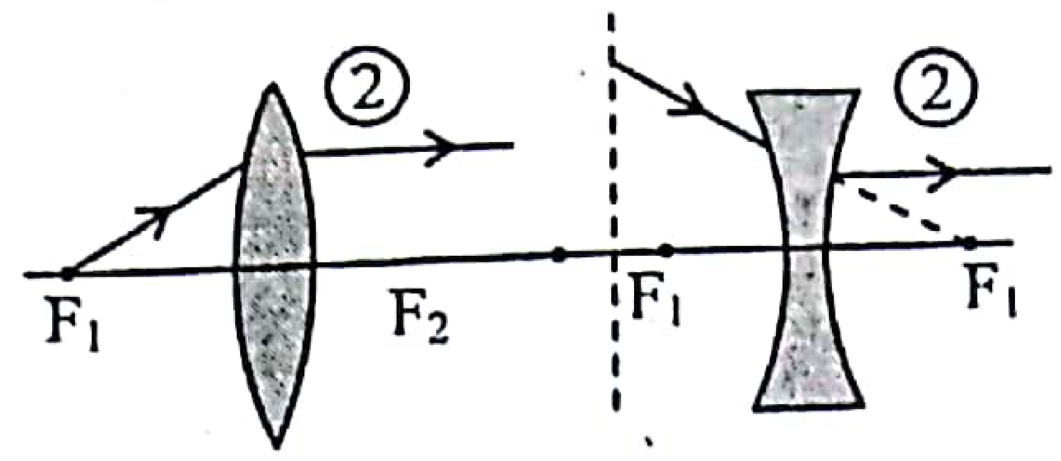
3.7 Ray diagram :

Graphically we can locate the position of image for a given object by drawing any two of the following three rays.

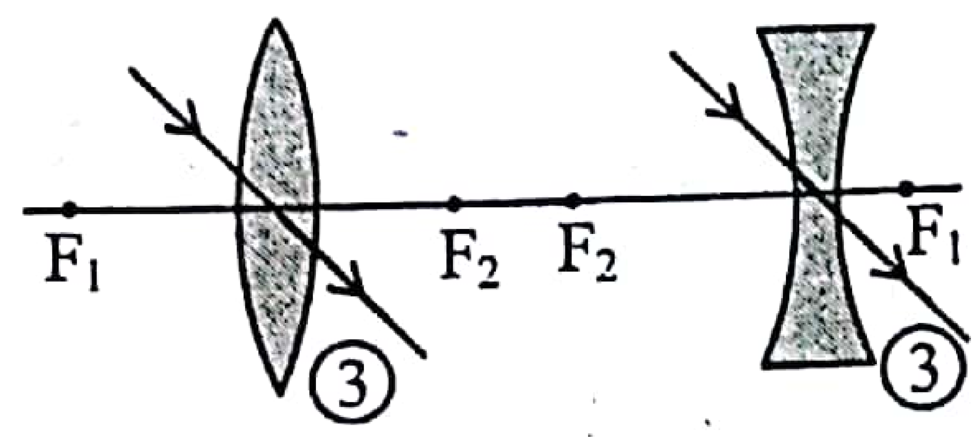
(A) A ray, initially parallel to the principal axis will pass (or appear to pass) through focus.



(B) A ray which initially passes (or appear to pass) through focus will emerge from the, lens parallel to the principal axis.



(C) A ray passing through the optical centre of the lens goes underviated as it passes through the lens region.



3.8 Position, size and nature of the images formed by a lens :

(A) Convex lens :

Suppose a real object is placed at a distance x from a convex lens of focal length f_o.The lens formulae may be modified as

$$\frac{1}{v}-\frac{1}{-x}=\frac{1}{f_o}$$

$$\Rightarrow v=\frac{xf_o}{x-f_o} \text{ And, lateral magnification,}$$

$$m=\frac{v}{u}=\frac{v}{-x}=-\frac{f_o}{x-f_o}$$

Note :

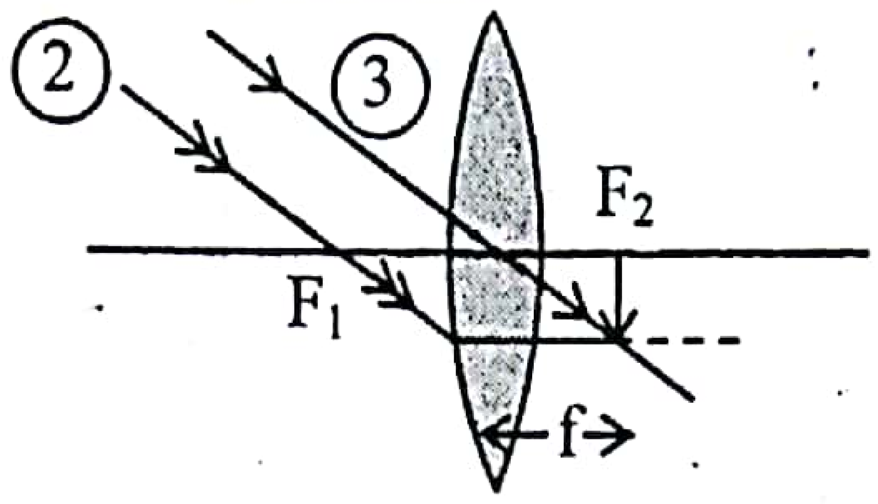
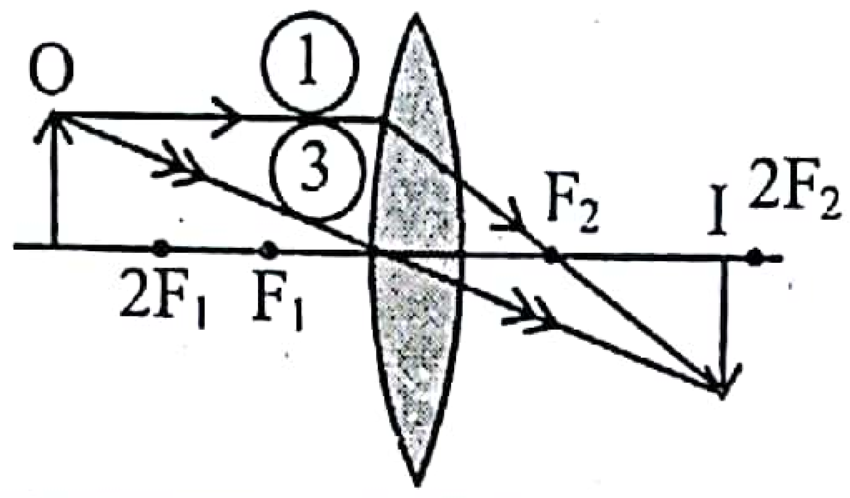
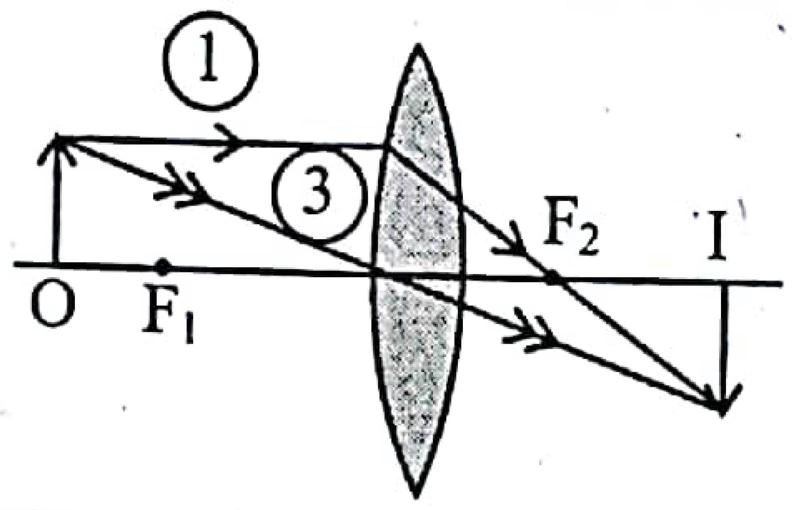
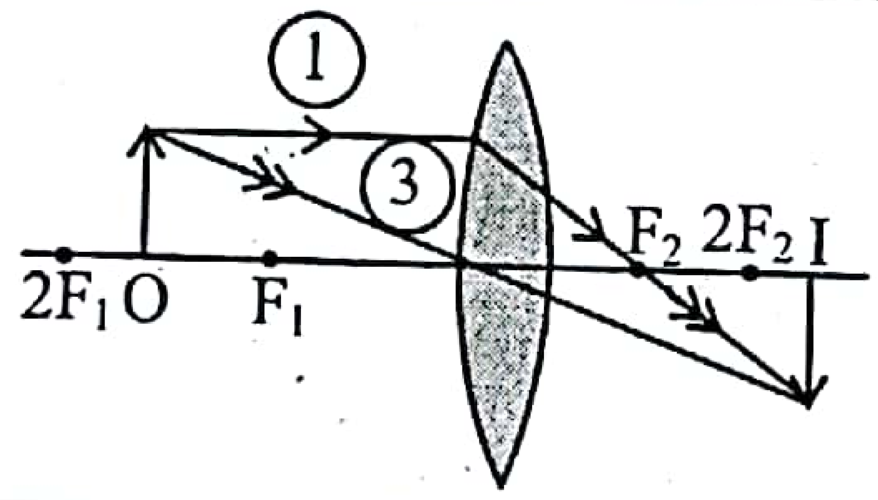
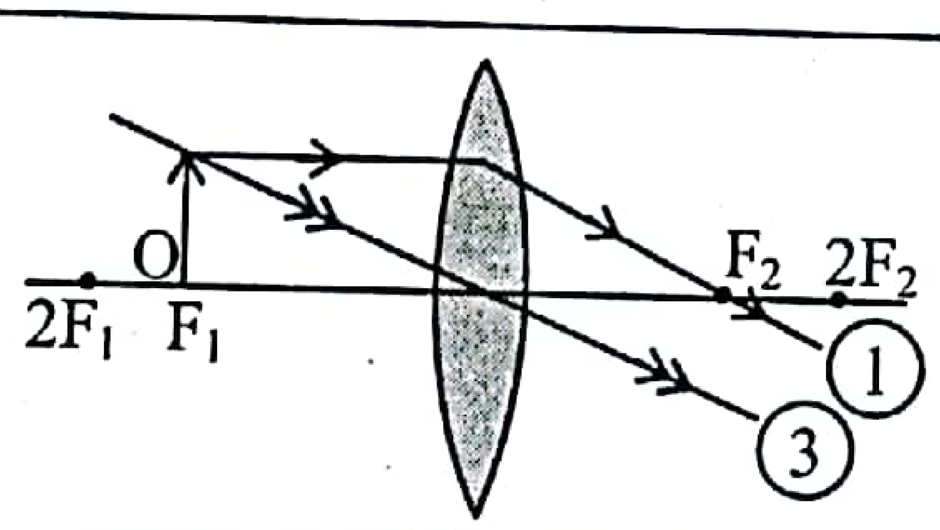
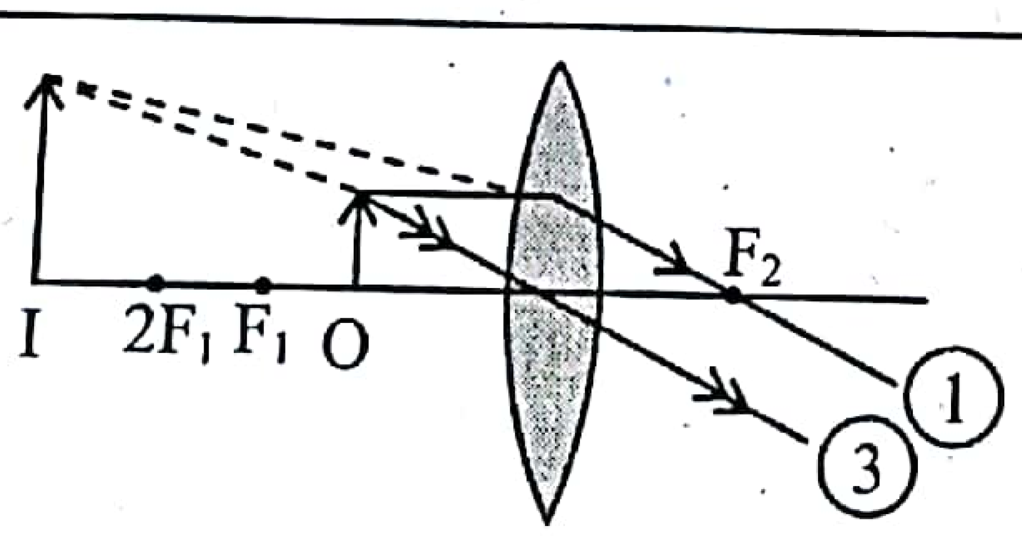
(1) A convex lens will form a real image for a real object when the object is placed beyond focus ($x > f_o$)

(2) When the object comes within focus i.e., $x < f_o$, then a virtual image is formed for the real object.

(3) The real image formed is always inverted while the virtual image is always erect.

(4) Anything (object or image) which is farther from the lens is always larger.

(a) For convergent or convex lens

S.No.	Position of Object	Position of image	Ray – Diagram	Nature (size)
1.	At infinity	At focus		Real, inverted, [Highly Diminished] ($m << -1$)
2.	Between ∞ and 2F	Between F & 2F		Real, inverted, [Diminished ($m < -1$)]
3.	At 2F	At 2F		Real, inverted, [Equal ($m = -1$)]
4.	Between 2F & F	Between 2F & ∞		Real, inverted, [Enlarged ($m > -1$)]
5.	At F	At ∞		Real, inverted, [Highly Enlarged ($m >> -1$)]
6.	Between F and P	Between ∞ & O on same side		Virtual, erect [Enlarged ($m > +1$)]

(B) Concave lens :

Suppose a real object is placed at a distance x in front of a concave lens of focal length f_0. Then the lens formulae can be modified as

$$\frac{1}{v} - \frac{1}{-x} = \frac{1}{-f_o} \Rightarrow v = \frac{-xf_o}{x+f_o}$$

And , lateral magnification,

$$m = \frac{v}{u} = \frac{v}{-x} = \frac{f_o}{x+f_o}$$

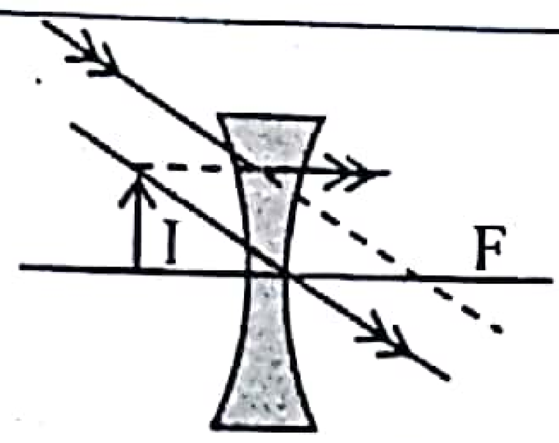
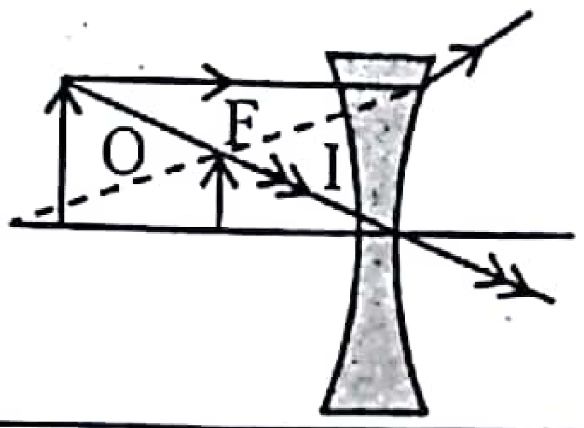
Note :

(1) A concave lens always form a virtual image for a real object .

(2) The image formed by a concave lens is always erect and diminished in size real object.

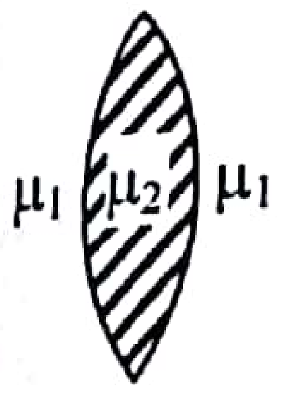
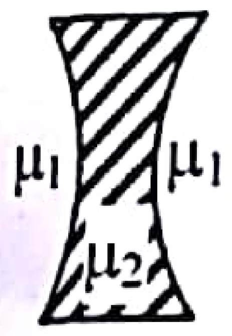
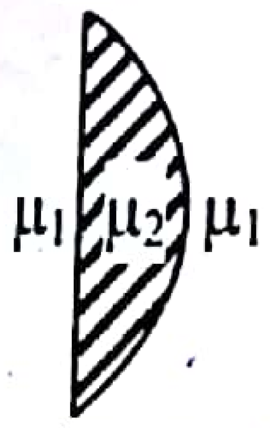
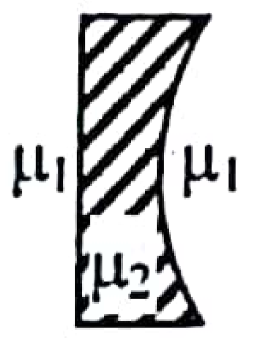
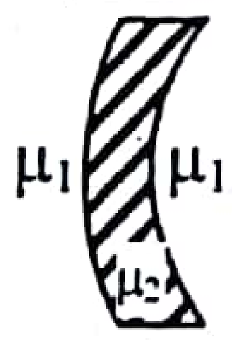
(3) A concave lens can form a real image if the object is virtual.

(a) **For Divergent or Concave Lens**

S. No.	Position of object	position of image	Ray-Diagram	Nature of image	Size
1.	At infinity	At F		Virtual, erect	Highly diminished ($m << +1$)
2.	In front of lens	Between F & optical centre		Virtual, erect	Diminished ($m < +1$)

3.9 Nature of varios lenses depending upon their surroundings :

If μ_1 is the R.I of surrounding and μ_2 is that of lens then

Shape of lens	Nature of lens	
	For $\mu_1 < \mu_2$	For $\mu_1 > \mu_2$
	Converging	Diverging
	Diverging	Converging
	Converging	Diverging
	Diverging	Converging
	Depends on the radius of curvature of the first and second surface	

3.10 Power of a lens :

When focal length is written in metre then

$P = \frac{\mu}{f}$ D is known as the power of the lens.

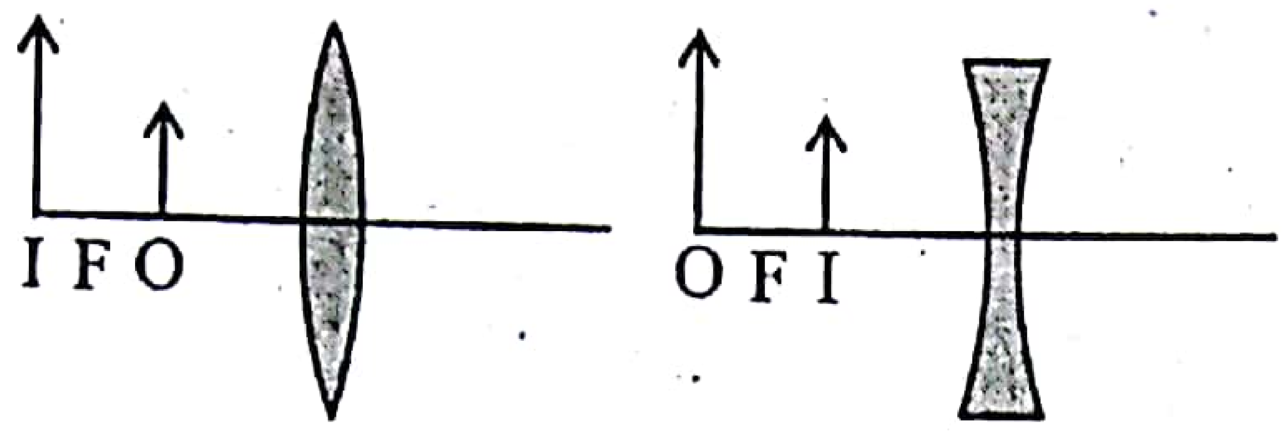
Where D is (diopter) unit of power and μ is the refractive index of the medium in which the lens is placed.

3.11 Representation of lens :

Converging lens is represent as

while diverging lens in represented as

3.12 Discussion



(A) (B)

(A) For real extended objects if the image formed by a single lens is erect (i.e. m is positive) it is always virtual. In this situation if the image is enlarged the lens is converging (i.e. convex) with object between focus and optical centre and if diminished the lens is diverging (i.e. concave) with image between focus and optical centre.

(B) As every part of a lens forms complete image, if a portion (say lower half) is obstructed (say covered with black paper) full image will be formed but brightness i.e. intensity will be reduced (to half). Also if a lens is painted with black strips and a donkey is seen through it, the donkey will not appear as a zebra but will remain a donkey with reduced intensity.

(C) If L is the distance between a real object and its real image formed by a lens, then as

$L = (|u| + |v|) = (\sqrt{u} - \sqrt{v})^2 + 2\sqrt{uv}$

So L will be minimum when

$(\sqrt{u} - \sqrt{v})^2 = \min = 0$ i.e. $u = v$

On substituting u = – u and v = +u in lens formula, we get

$\frac{1}{u} - \frac{1}{-u} = \frac{1}{f}$ i.e. $u = 2f$

So that $(L)_{min} = 2\sqrt{2f \times 2f} = 4f$

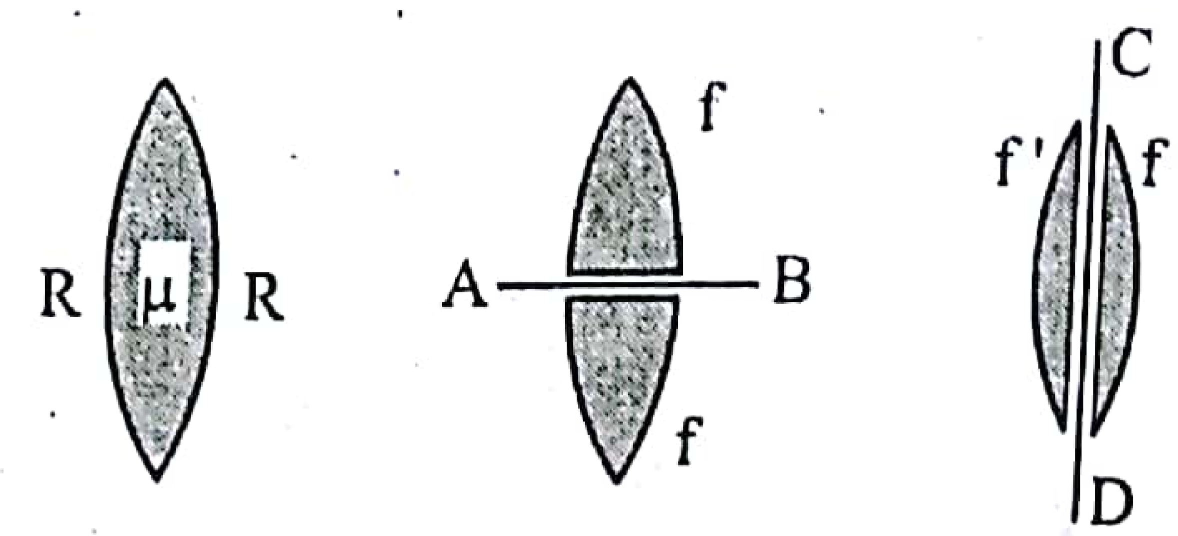
[as L_{min}, $u = v$]

i.e. the minimum distance between a real object and its real image formed by a single lens is 4f.

(D) If an object is moved at constant speed towards a convex lens from infinity to focus, the image will move slower in the beginning and faster later on, away from the lens. This is because in the time the object moves from infinity to 2F, the image will move from F to 2F and when the object moves from 2F to F, the image will move from 2F to infinity. At 2F the speed of object and image will be equal. It can be shown that in case of lens, speed of image.

$$V_i = V_0 \left[\frac{f}{u+f}\right]^2$$

(E) If an equi-convex lens of focal length f is cut into two equal parts by a horizontal plane AB, then as none of μ, R_1 and R_2 will change, the focal length of each part be equal to that of initial lens, i.e.



$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{-R}\right] = \frac{2(\mu - 1)}{R}$$

However in this situation as light transmitting area of each part becomes half of initial, so intensity will be reduced to half and aperture to $(1/\sqrt{2})$ times of its initial value

[as I ∝ $(\text{Aperture})^2$].

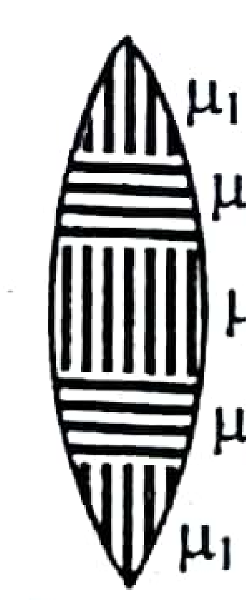
However, if the same lens is cut into two equal parts by a vertical plane CD, the focal length of each part will become.

$$\frac{1}{f'} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right] = \frac{(\mu - 1)}{R} = \frac{1}{2f}$$

i.e., $f' = 2f$

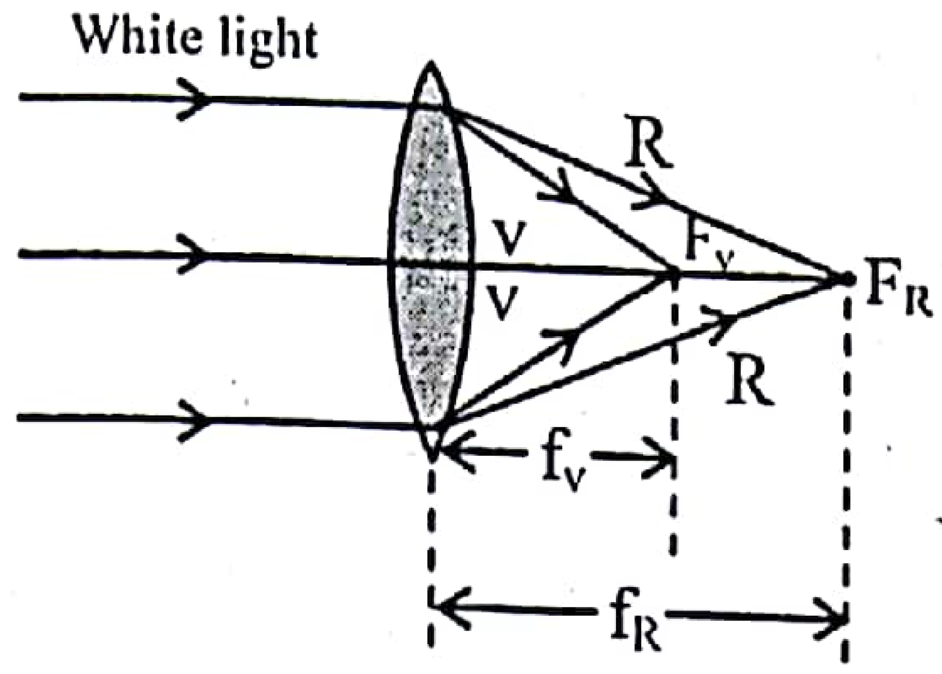
i.e., focal length of each part will be double of initial value. In this situation as the light transmitting area of each part of lens remains equal to initial, intensity and aperture will not change.

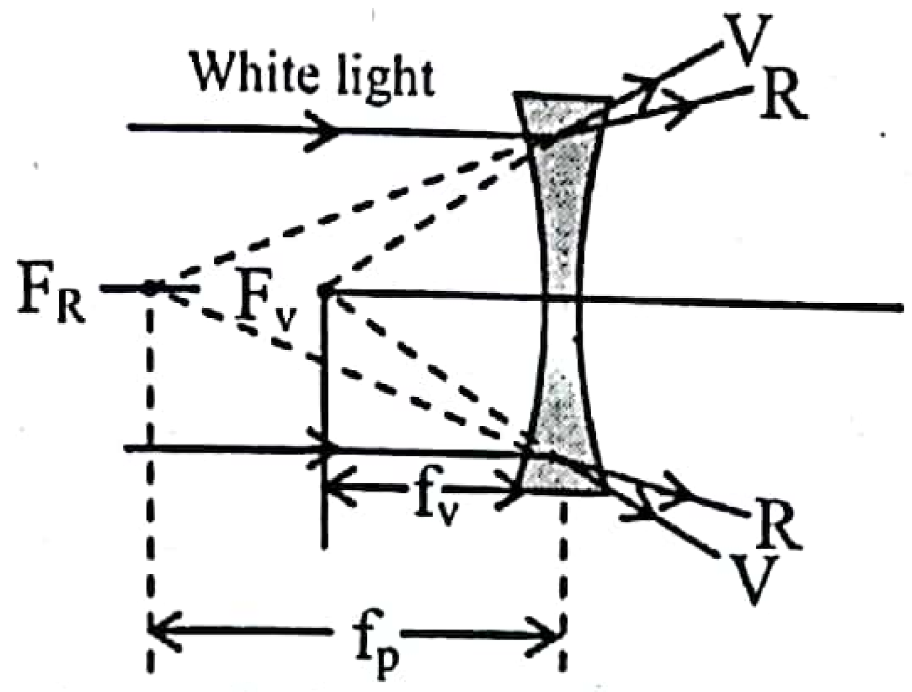
(F) If a lens is made of a number of layers of different refractive indices as shown in figure for a given wavelength of light it will have as many focal



lengths or will form as many images as there are μ's as $\frac{1}{f} \propto (\mu - 1)$

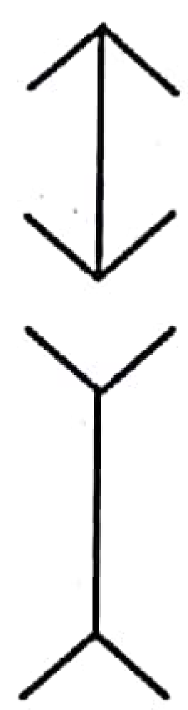
(G) As focal length of a lens depends on μ, i.e. $(1/f) \propto (\mu - 1)$, the focal length of a given lens is different for different wavelengths and maximum for red and minimum for violet whatever be the nature of lens.





(H) If a lens of glass (μ = 3/2) is shifted from air (μ = 1) to water (μ = 4/3) then as :

$$\frac{1}{f_A} = \left[\frac{3/2}{1} - 1\right] K \text{ and}$$



$$\frac{1}{f_V} = \left[\frac{(3/2)}{(4/3)} - 1\right] K$$

$$\text{with } K = \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

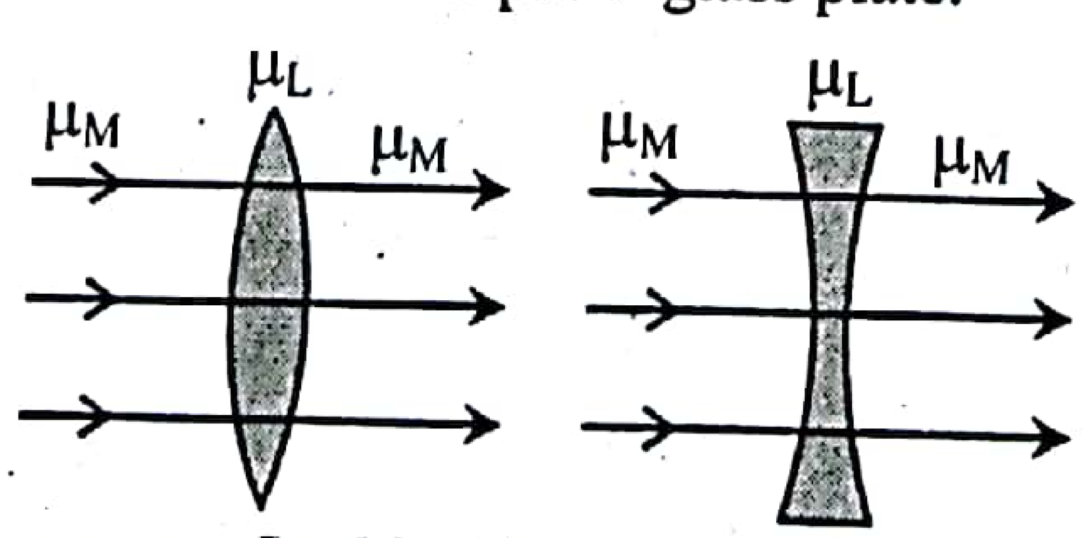
$$\frac{f_W}{f_A} = \left[\frac{8}{K}\right] \times \left[\frac{K}{2}\right] \text{ i.e. } f_W = 4f_A$$

i.e. focal length of a lens in water becomes 4 times of its value in air.

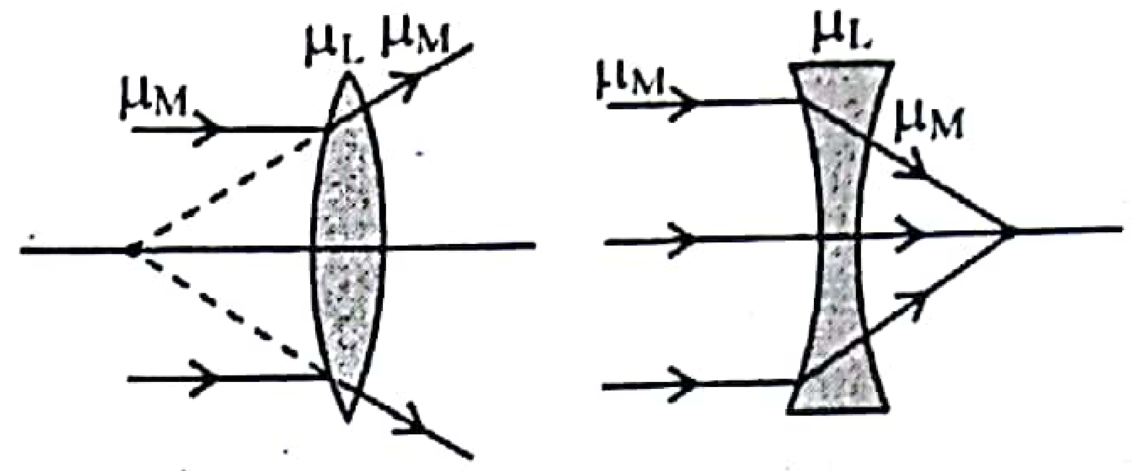
(I) If a lens is shifted from one medium to the other, depending on the refractive index of the lens and medium, the following three situations are possible.

(i) $\mu_M < \mu_L$ **but** μ_M **increases :** In this situation $\mu = (\mu_L/\mu_M)$ will remain greater than unity but will decrease and as $(1/f) \propto (\mu - 1)$, $(1/f)$ will decrease i.e., f will increase (without change in nature of lens)

(ii) $\mu_M = \mu_L$ **:** In this situation $\mu = (\mu_L/\mu_M) = 1$, so that $(1/f) \propto (\mu - 1) = 0$, i.e., $f = \infty$, i.e., lens will neither converge nor diverge but will behave as a plane glass plate.



(iii) $\mu_M > \mu_L$**:** In this situation $\mu = (\mu_L / \mu_M) < 1$, so in **Lens-maker's formula** sign of f and hence nature of lens will change, i.e. a converging lens will behave as divergent and vice-versa.



4. COMBINATION OF LENSES

4.1 When lenses are in contact with each other

When several lenses are kept co-axially , the image formation is considered one after another in steps. The image formed by the lens facing the object serves as an object for the next lens, the image formed by the second acts as an object the third and so on .

(A) Net magnification , $m = m_1 \times m_2 \times m_3 \times .....$

(B) If thin lenses are kept close together with their principal axis coinside then,

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...}$$

$$\boxed{\text{or } P = P_1 + P_2 + P_3 + ...}$$

Note :

1. If the two thin lenses are separated by a distance ' d ' then, $P = P_1 + P_2 - dP_1P_2$

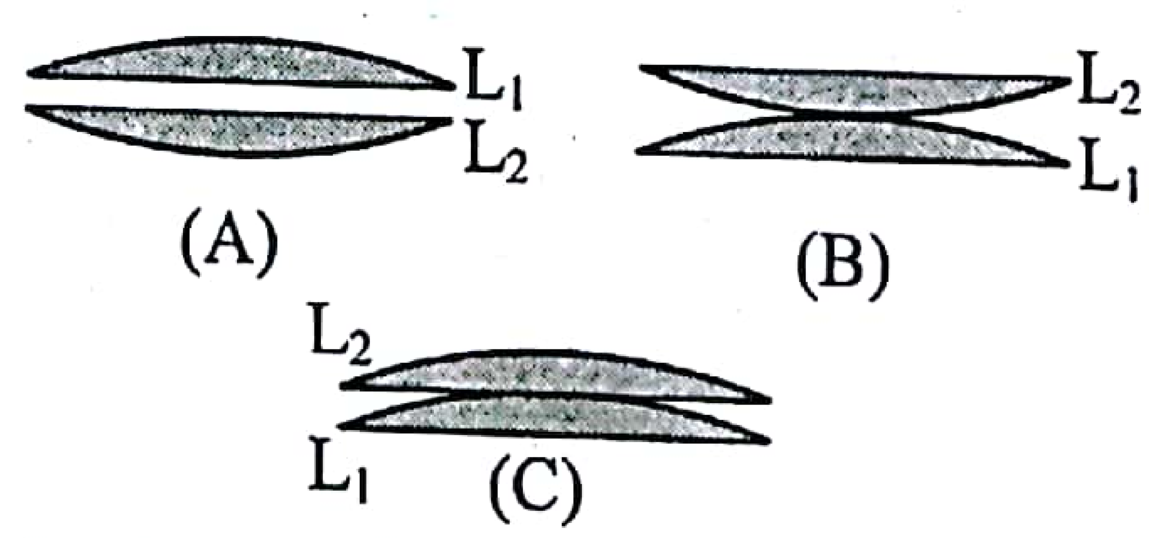
2. If a lens of focal length f is divided into two equal parts as shown in Fig.(A) and each part has a focal length f ' then as

$$\frac{1}{f} = \frac{1}{f'} + \frac{1}{f'} \qquad \text{i.e., } f' = 2f$$

i.e., each part will have focal length 2f.

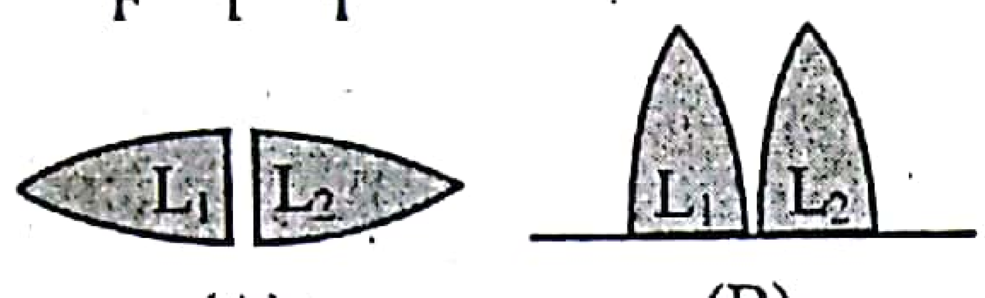
Now if thses parts are put in contact as in Fig. (B) or (C) the resultant focal length of the combination will be

$$\frac{1}{F} = \frac{1}{2f} + \frac{1}{2f} \qquad \text{i.e., } F = f\,(= \text{initial value})$$



3. If a lens of focal length f is cut in two equal parts as shown in figure (A) each part will have focal length f. Now if these parths are put in contact as shown in figure (B) the resultant focal length will be.

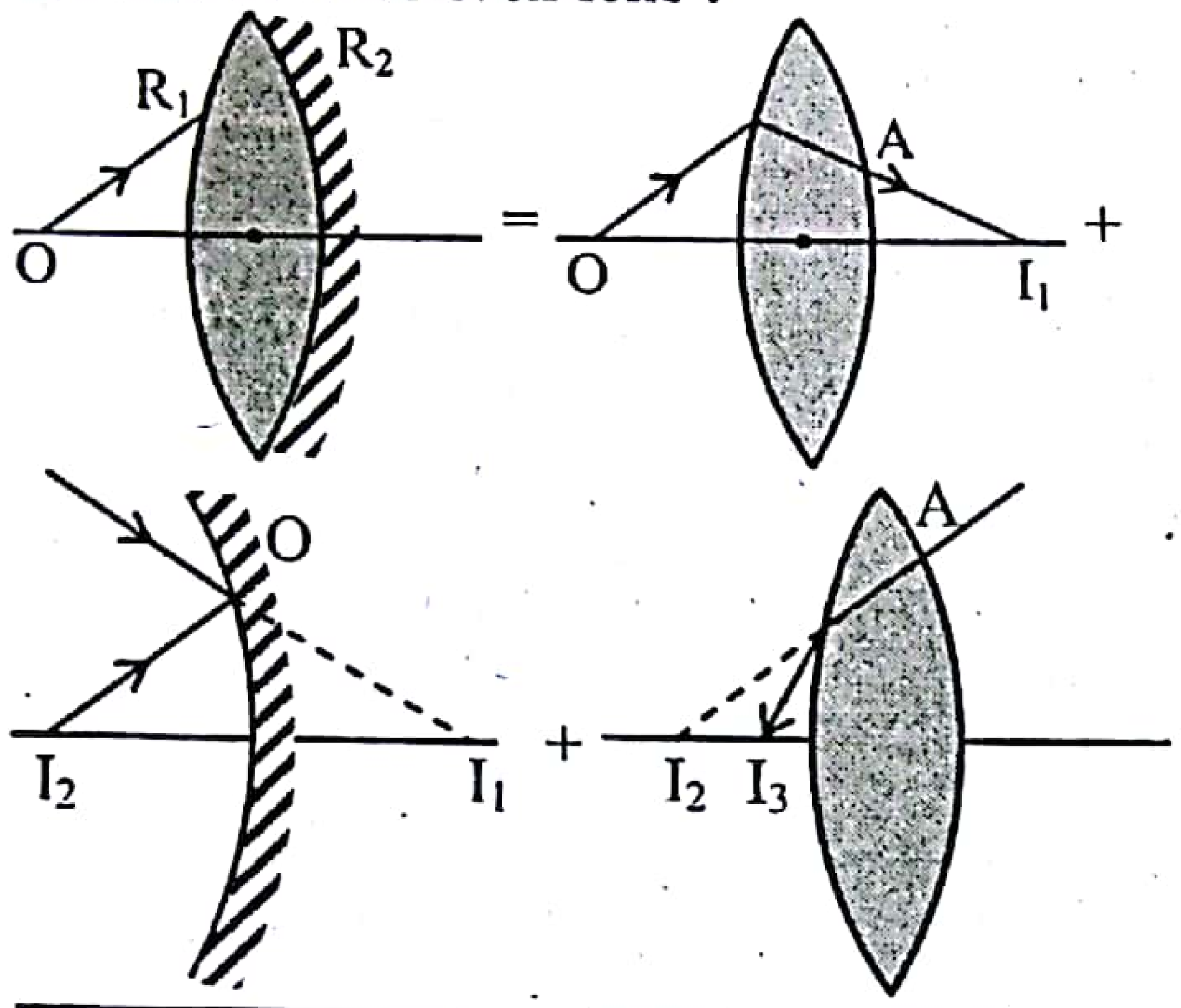
$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f} \quad \text{i.e. } F=(f/2)$$



(A) (B)

5. LENS WITH ONE SURFACE SILVERED

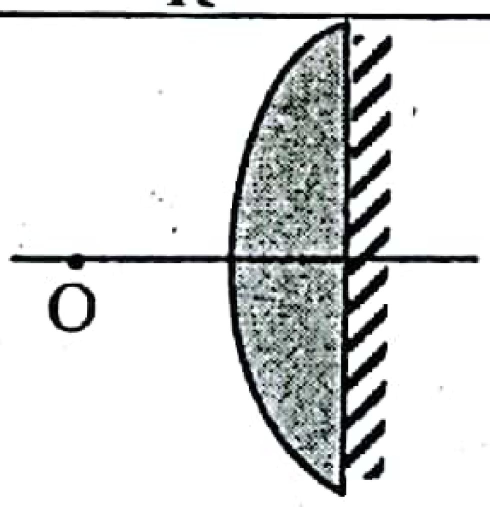
A. Silvered convex lens :



$$P=\frac{2}{R}[2\mu-1] \qquad f=\frac{-R}{2[2\mu-1]}$$

B. When the plane surface is silvered:

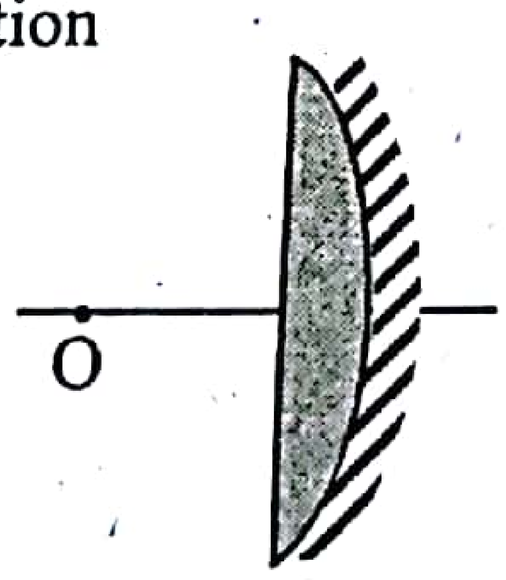
i.e., $$P=2\frac{(\mu-1)}{R}+0=\frac{2(\mu-1)}{R} \quad ...(1)$$



$$f=-\frac{1}{P}=-\frac{R}{2(\mu-1)} \quad ...(2)$$

C. When the curved surface is silvered :

Here the object is in front of plane surface In this situation



i.e., $$P=\frac{2(\mu-1)}{R}+\frac{2}{R}=\frac{2\mu}{R} \quad .....(3)$$

So $$f=-\frac{1}{P}=-\frac{R}{2\mu} \quad .....(4)$$

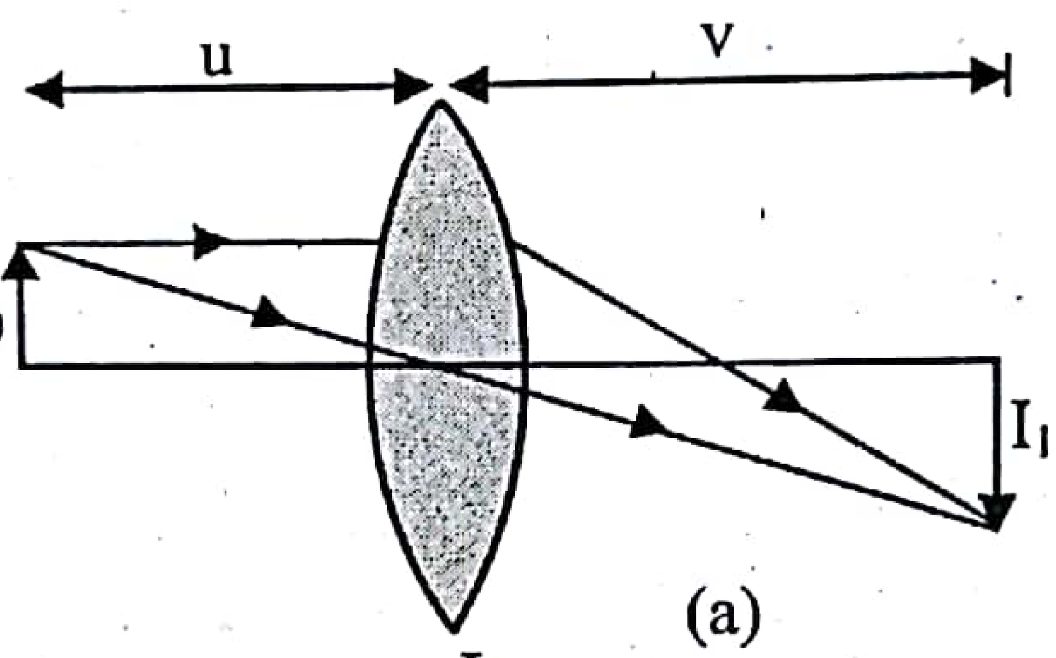
6. DISPLACEMENT METHOD FOR FINDING THE FOCAL LENGTH/POWER OF A CONVEX LENS

(i) In this method, the distance between the object and the screen must be greater than $4f$, where f is the focal length of the convex lens.

(ii) The image on the screen can be formed corresponding to two different positions of the lens.

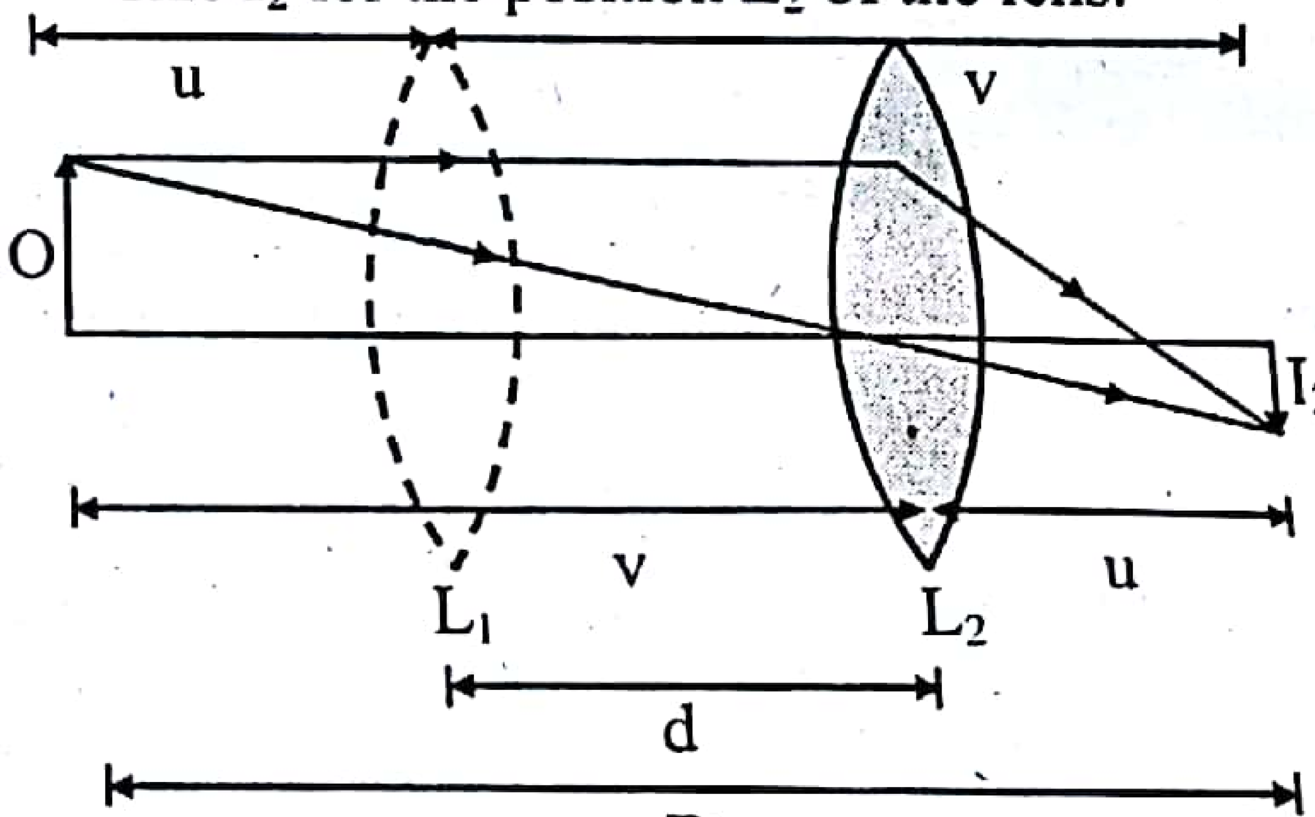
(iii) Figure (a) shows the magnified image of size I_1 for the position L_1 of the lens.

$$m_1=\frac{I_1}{O}=\frac{v}{u} \quad .....(1)$$



(a)

(iv) Figure (b) shows the diminished image of size I_2 for the position L_2 of the lens.



(b)

$$m_2=\frac{I_2}{O}=\frac{u}{v} \quad ......(2)$$

From (1) and (2),

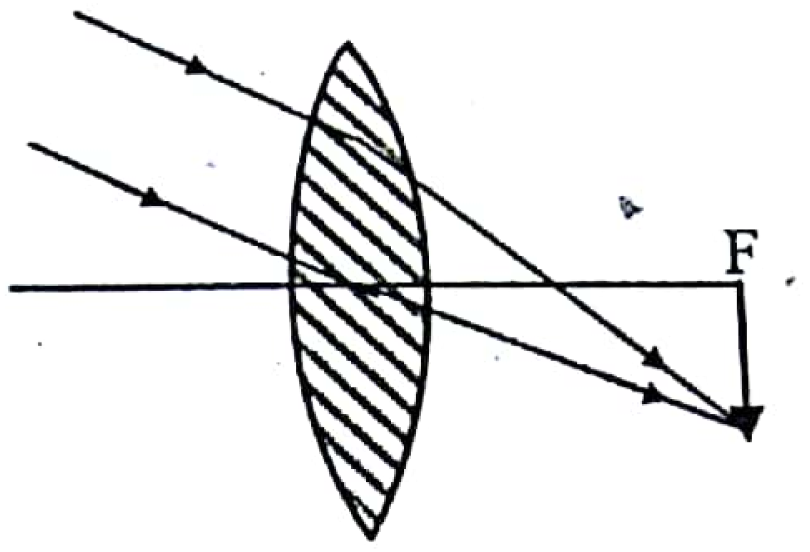
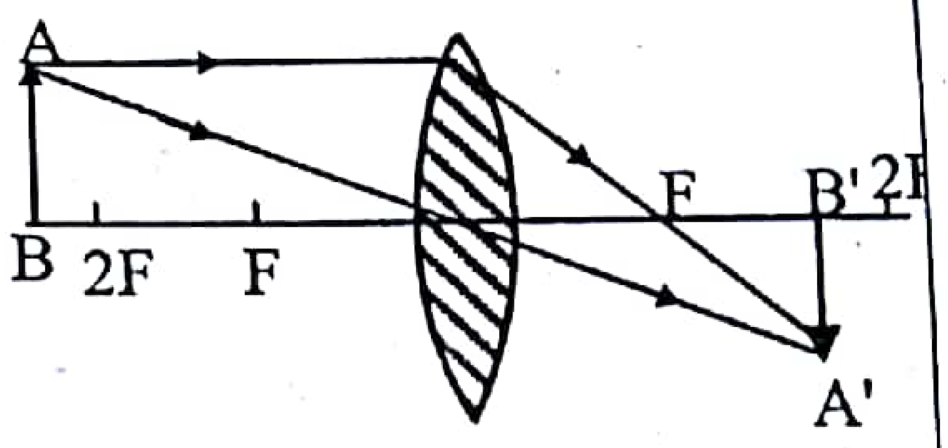
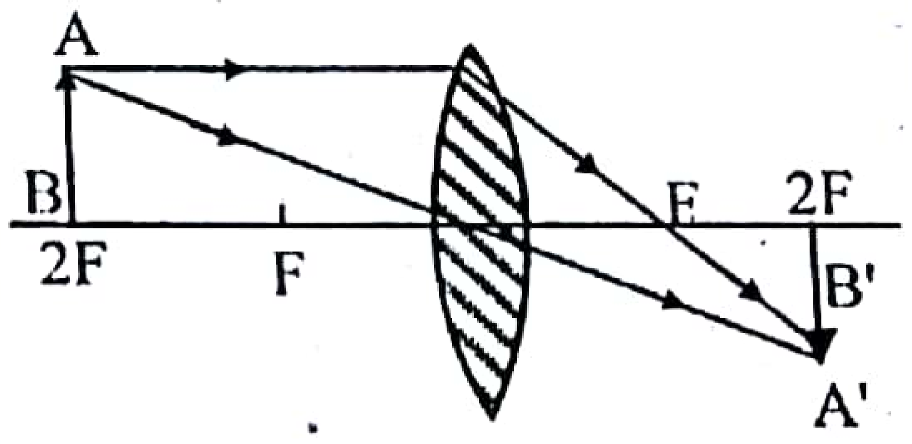
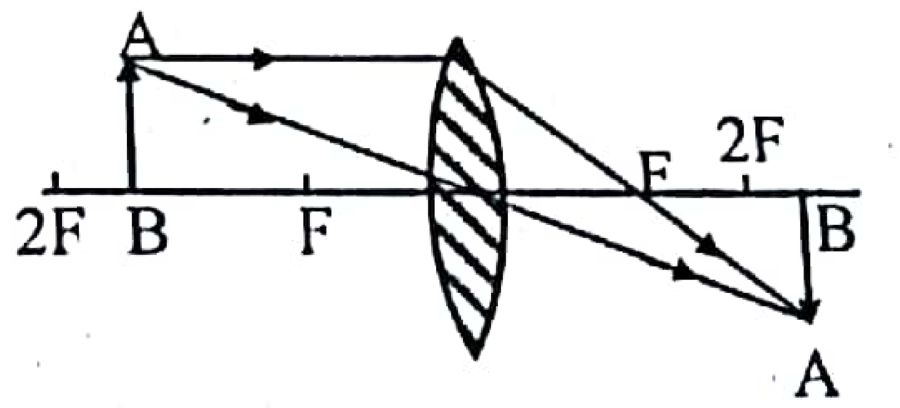
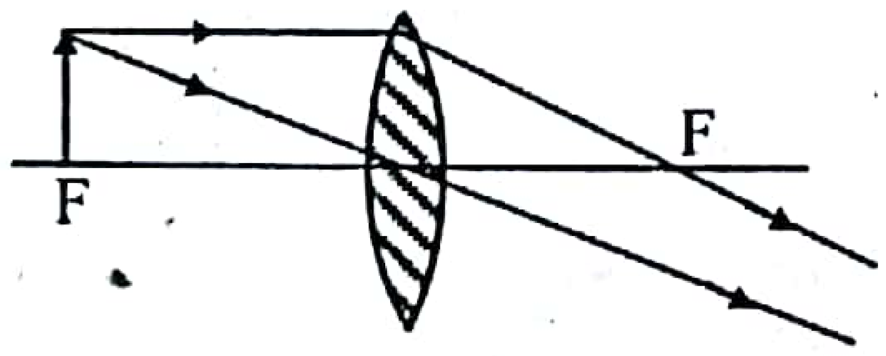
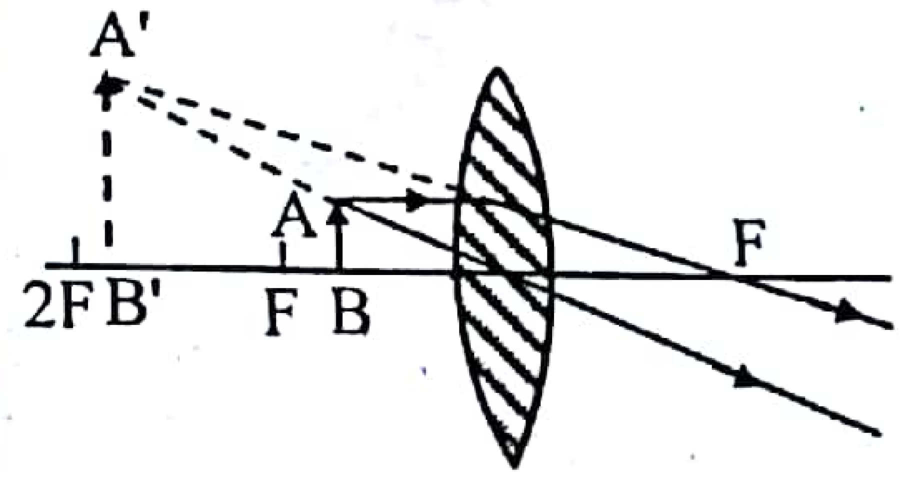
$$\frac{I_1I_2}{O^2}=\frac{v}{u}\times\frac{u}{v}=1 \text{ or } O=\sqrt{I_1I_2}$$

Also, $$\frac{I_1}{I_2}=\frac{v}{u}\times\frac{v}{u}=\frac{v^2}{u^2}=\frac{(D+d)^2}{(D-d)^2}$$

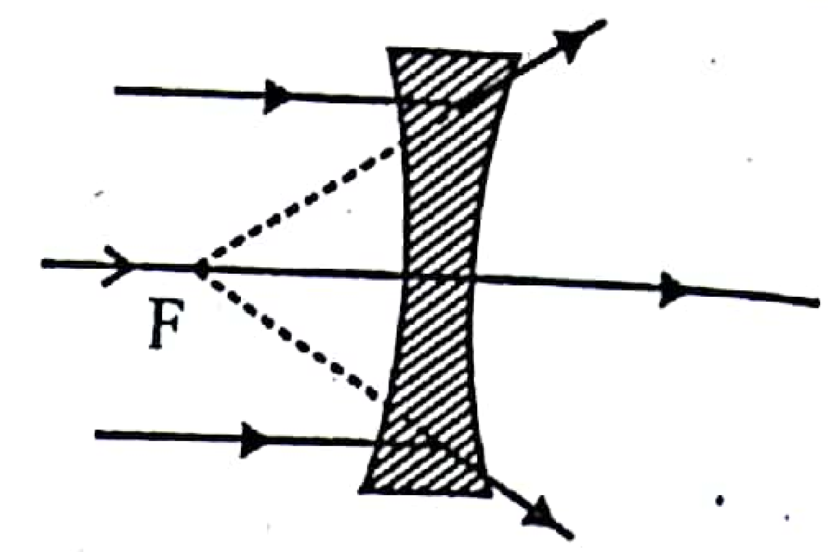
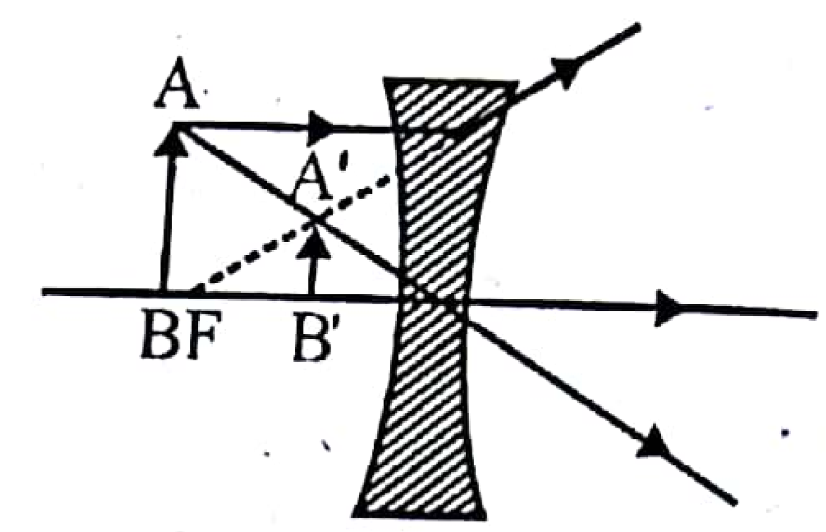
(v) $$\frac{m_1}{m_2}=\frac{v^2}{u^2}$$

(vi) $$f=\frac{D^2-d^2}{4D} \quad \text{Also, } P=\frac{4D}{D^2-d^2}$$

Convex Lens

Position of object	Position of image	Real/ virtual	Inverted/ erect	Magnification and size of image	Sign of magnification	RAY DIAGRAM
at infinity ($u = \infty$)	at focus ($v = f$)	real	inverted	$m < 1$ greatly diminished	negative	
beyond 2f($u > 2f$)	between f and 2f ($f < v < 2f$)	real	inverted	$m < 1$ diminished	negative	
at 2f ($u = 2f$)	at 2f ($v = 2f$)	real	inverted	$m = 1$ same size	negative	
between f and 2f ($f < u < 2f$)	beyond 2f($v > 2f$)	real	inverted	$m > 1$ magnified	negative	
at f($u = f$)	at infinity ($v = \infty$)	real	inverted	$m = \infty$ magnified	negative	
between optical centre and focus ($u < f$)	at a distance greater than the object distance and on the same side as object ($v > u$)	virtual	erect	$m > 1$ magnified	positive	

Concave Lens

at infinity ($u = \infty$)	at focus ($v = f$)	virtual	erect	$m < 1$ diminished	positive	
between infinity and optical centre	between optical centre and focus	virtual	erect	$m < 1$ diminished	positive	

7. OPTICAL INSTRUMENTS

7.1 Definition

Optical instruments are used primarily to assist the eye in viewing an object.

7.2 Types of Instruments

Depending upon the use , optical instruments can be categorised in the following way :

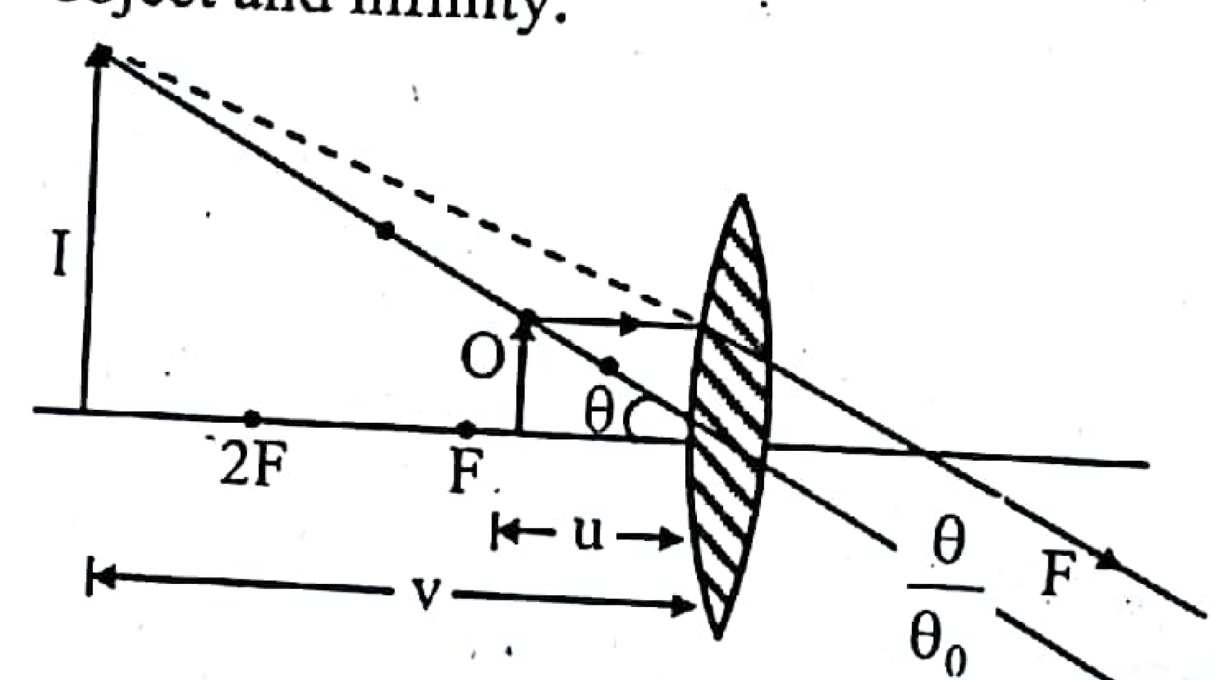
OPTICAL INSTRUMENT
- Camera
  - Pinhole
  - Lens
- Projector
- Telescope
  - Astronomical
    - Refracting
    - Reflecting
  - Terrestrial
    - Gallalian
    - Binocular
- Microscope
  - Simple
  - Compound

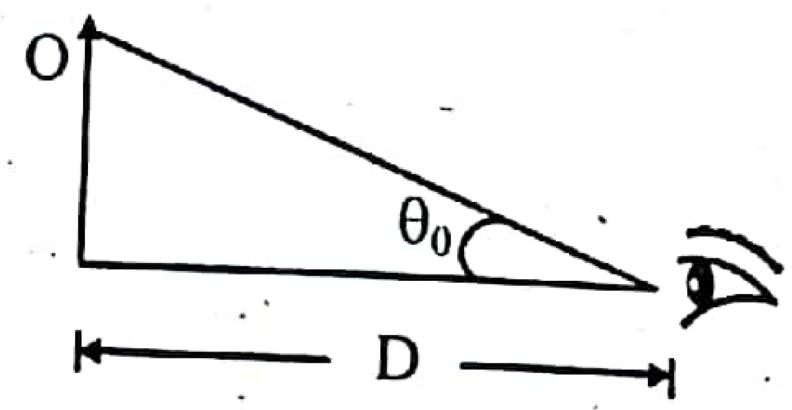
8. MICROSCOPE

It is an optical instrument used to increase the visual angle of near objects which are too small to be seen by naked eye. Microscopes are of two types viz. simple microscope and compound microscope.

8.1 Simple Microscope :

It is also known as magnifying glass or **magnifier** and consists of a convergent lens with object between its focus and optical centre and eye close to it. The image formed by it is erect, virtual, enlarged and on same side of lens between object and infinity.





Here

Magnifying power

$$= \frac{\text{visual angle with instrument}}{\text{Maximum visual angle for unaidedeye}} = \frac{\theta}{\theta_0}$$

Now, $\theta = \frac{h_i}{v} = \frac{h_0}{u}$ with, $\theta_0 = h_0/D$

$$\text{M.P.} = \frac{\theta}{\theta_0} = \frac{h_0}{u} \times \frac{D}{h_0} = \frac{D}{u}$$

Now, two possibilities are there :

(A) Image is at infinity (far point)

If $v = \infty$ $u = f$ (from lens formula)

So, $\text{M.P.} = \frac{D}{u} = \frac{D}{f}$.

Note : Here parallel beam of light enters the eye i.e., eye is least strained.

(B) Image is at D (Near point)

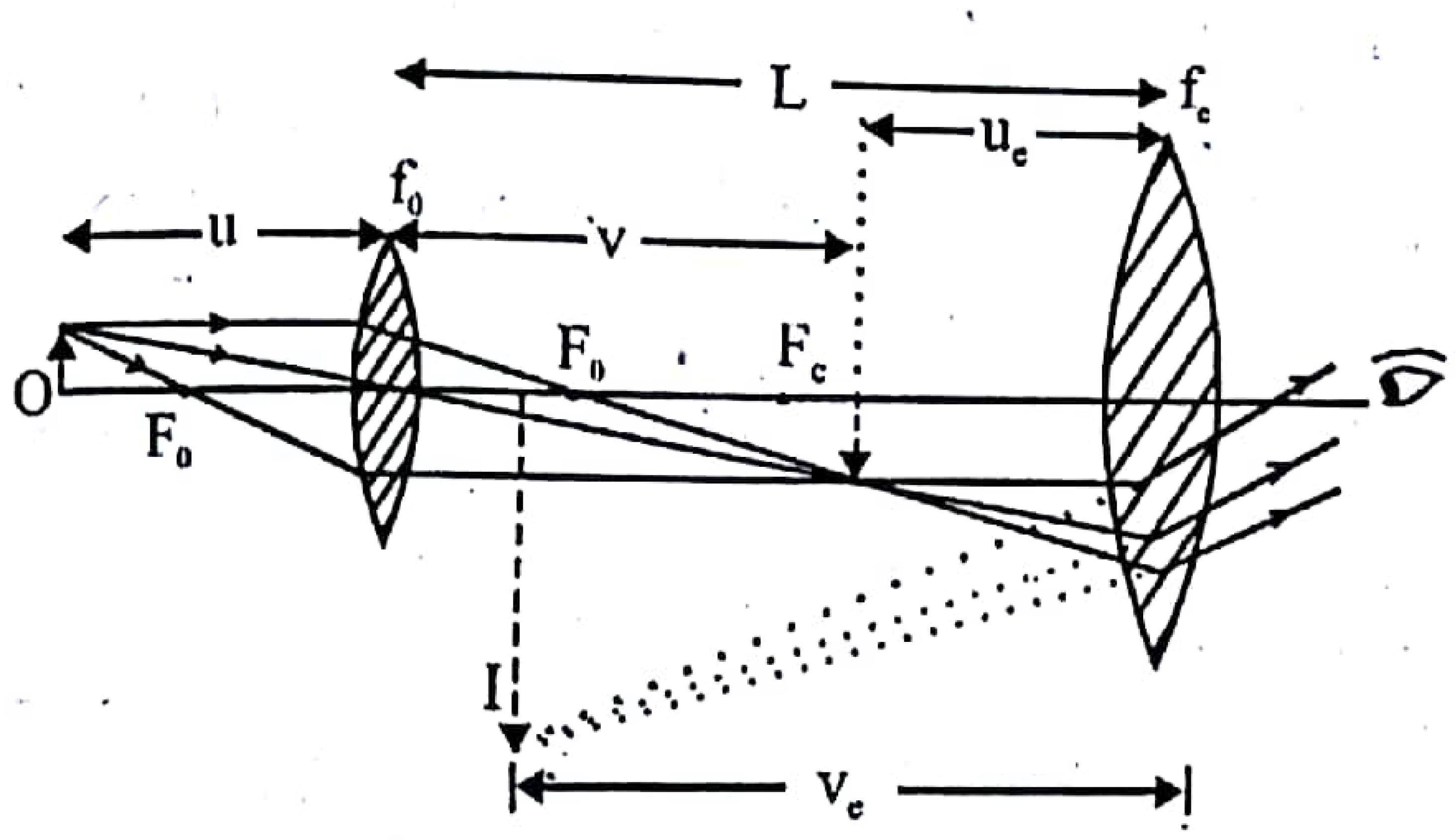
In this situation $v = -D$, so that, $\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$

or, $\frac{D}{u} = 1 + \frac{D}{f}$

So, M.P. $= 1 + \frac{D}{f}$

Note : Here final image is closest to eye i.e., eye is under maximum strain.

8.2 Compound Microscope



It consists of two convergent lenses of short focal lengths and apertures arrangedco-axially.

Lens f_0 is the **objective** or **field lens** and f_e is the **eye-piece** or **ocular**. Objective has smaller aperture and focal length than eye-piece. The separation between objective and eye-piece can be varied.

$$\text{Magnifying Power} = \frac{\theta}{\theta_0} = \frac{h_i}{u_e} \times \frac{D}{h_0} = \frac{h_i}{h_0} \times \frac{D}{u_e}$$

But for objective, $m = \frac{v}{u}$ i.e., $\frac{h_i}{h_0} = -\frac{v}{u}$

so, M.P. $= -\frac{v}{u}\left[\frac{D}{u_e}\right]$ where, $\mu_e + \mu = L$

Now two possibilities are there :

(A) Final image is at infinity (far point)

$\mu_e = f_e \Rightarrow$ M.P $= -\frac{v}{u}\left[\frac{D}{f_e}\right]$

where $L = u + f_e$

Note : A microscope is usually considered to operate in this mode unless stated otherwise.

(B) Final image is at D (near point)

For eye - piece . $v_e = D$,

$\Rightarrow \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$

$\Rightarrow \frac{1}{u_e} = \frac{1}{D}\left(1 + \frac{D}{f_e}\right)$

M.P. $= -\frac{v}{u}\left(1 + \frac{D}{f_e}\right)$ with $L = v + \frac{f_e D}{f_e + D}$

Note :

In case, $u \cong f_0$ and, $L = v + u_e \cong v$ so that

$|\text{M.P.}| \cong \frac{L}{f_0} \times \frac{D}{f_e}$

IMPORTANT POINTS

1. As **magnifying power** is negative, the image seen in a microscope is always truly inverted, i.e., left is turned right with upside down simultaneously.
2. **Resolving Power** : The minimum distance between two lines at which they are just distinct is called limit of resolution and reciprocal of limit of resolution is called resolving power.

R.P. $= \frac{1}{\Delta x} \propto \frac{1}{\lambda} = 2\mu\sin/\theta/\lambda$

9. TELESCOPE

It is an optical instrument used to increase the visual angle of distant large objects.

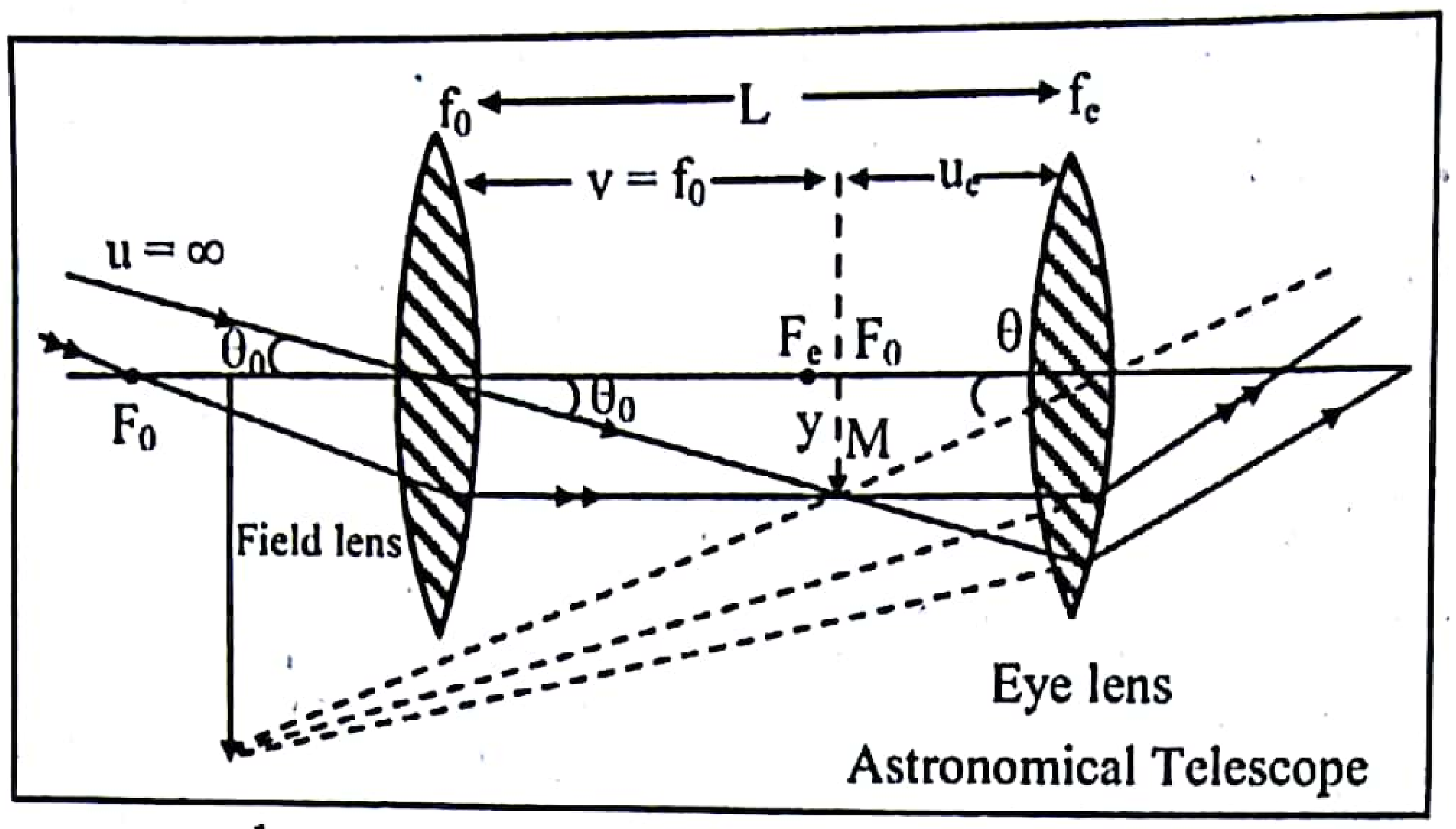
Telescopes mainly are of two types viz. astronomical and terrestrial.

9.1 Astronomical Telescope :

It consists of two converging lenses placed coaxially with objective having large aperture and a large focal length while the eye- piece having smaller aperture and focal length. The separation between eye- piece and objective can be varied.

Magnifying power

$= \frac{\text{visual angle with instrument}}{\text{visual angle for unaided eye}} = \frac{\theta}{\theta_0}$



Astronomical Telescope

$$\theta_0 = \frac{h}{f_0} \quad \& \quad \theta = h/(-u_e)$$

$$\Rightarrow \text{M.P.} = -\left[\frac{f_0}{u_e}\right] \qquad \text{with } L = f_0 + u_e$$

Now two possibilities are there

(A) Final image is at infinity (far point)

Here, $v = \infty \Rightarrow u_e = f_e$

So, $\text{M.P.} = -(f_0/f_e)$ with, $L = f_0 + f_e$

Note :

Usually , a telescope operates in this mode unless stated otherwise.

(B) Final image is at D (near point)

$$\text{Here, } v = D \Rightarrow \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$= \frac{1}{u_e} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$$

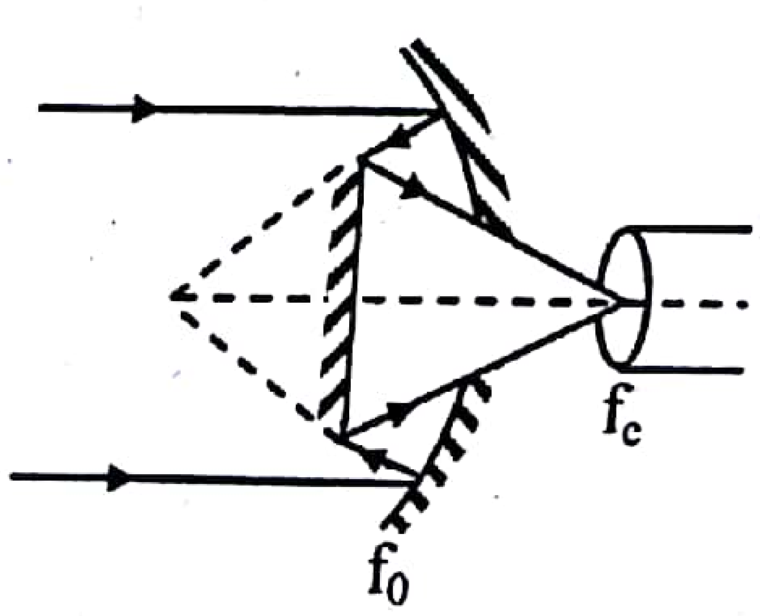
$$\text{So, M.P.} = -\frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right]$$

$$\text{with } L = f_0 + \frac{f_e D}{f_e + D}$$

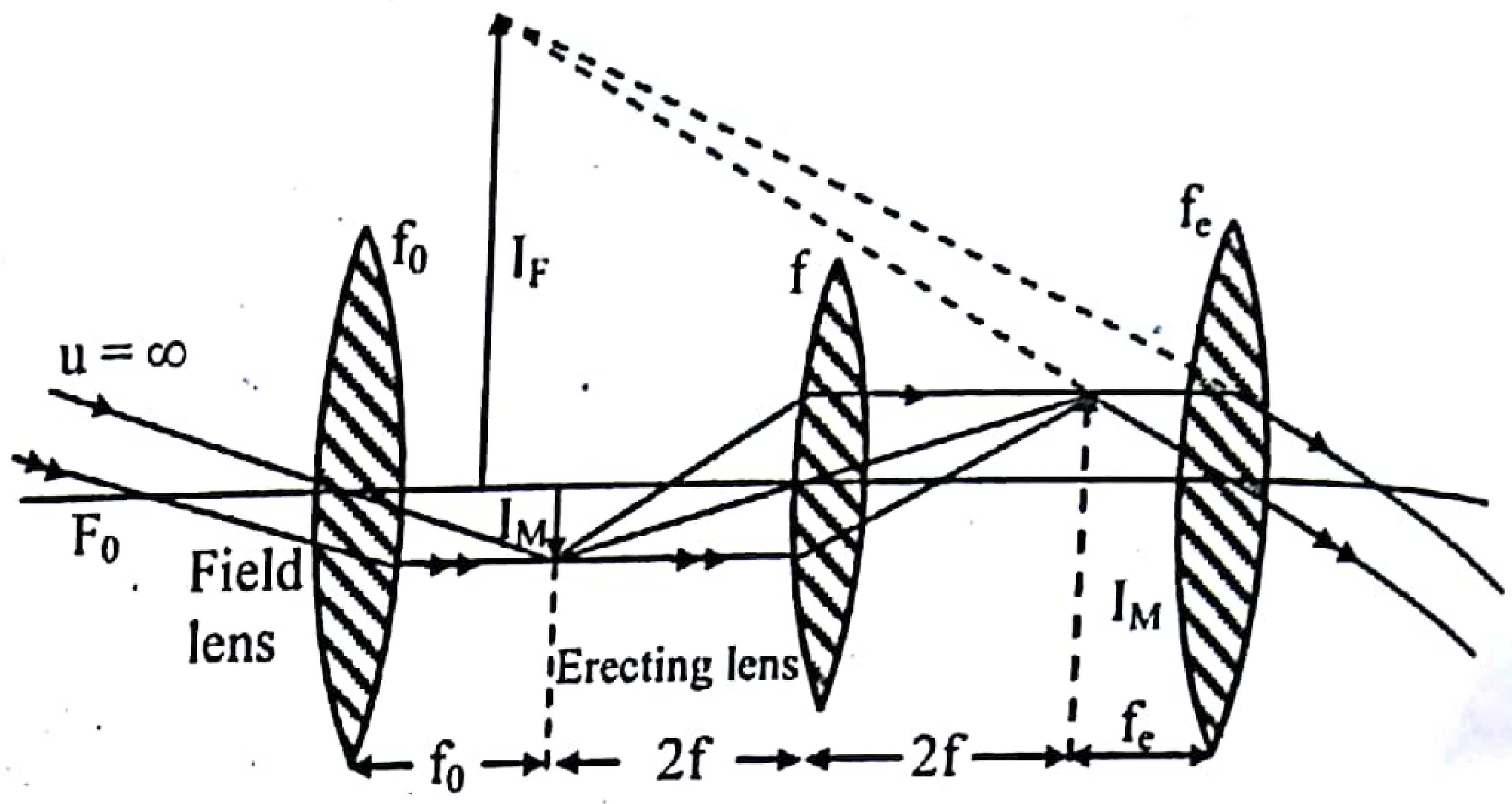
Note :

1. The above discussion is that of the **refracting telescope.**
2. **Reflecting Telescope :**

If the field lens of refracting telescope is replaced by a converging mirror, then the telescope becomes a reflecting one, where $\text{M.P.} = f_0/f_e$



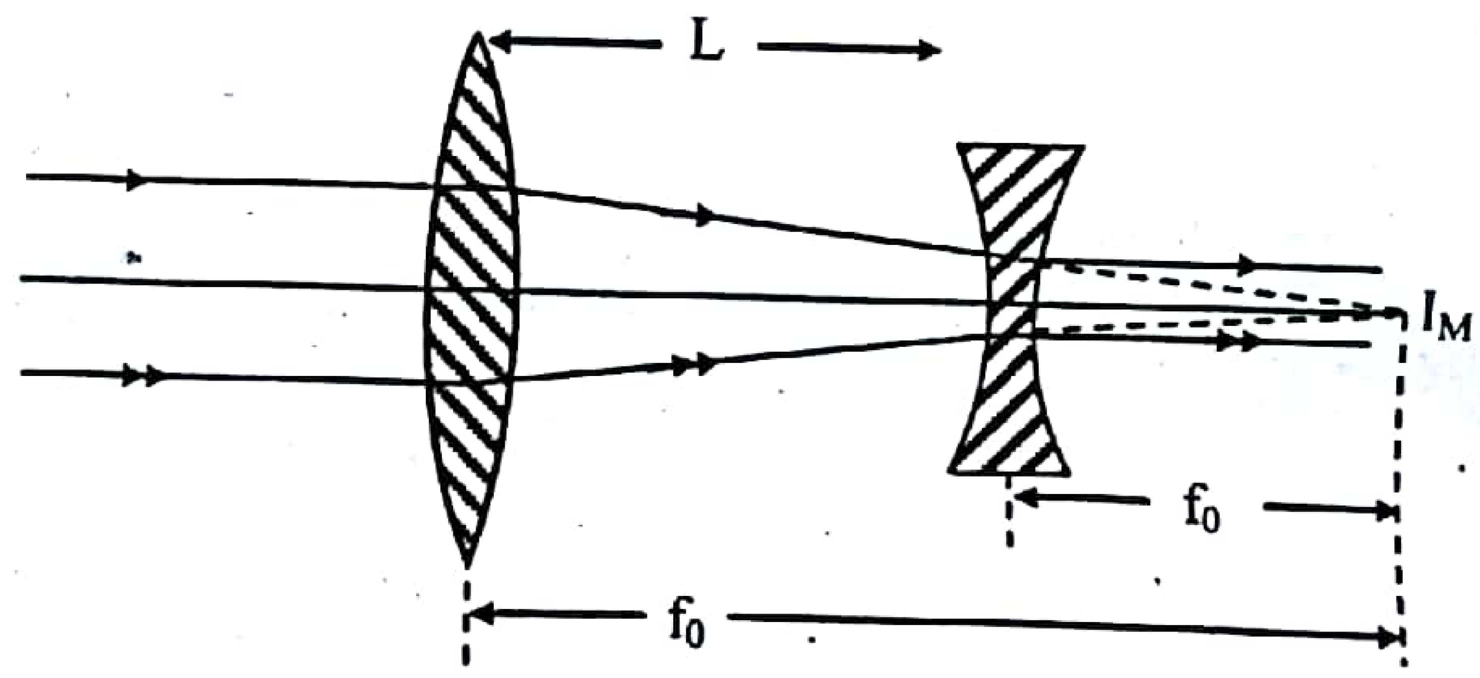
9.2 Terrestrial Telescope :



If a lens of short focal length f is placed at 2f from the intermediate image at a distance 2f on the other side of it and this image will act as an object for eye- lens which will produce erect image with respect to the object; this lens is called **erecting lens** and as for it m = – 1, the MP and length of telescope for relaxed eye will be

$$\text{M.P.} = -\frac{f_0}{f_e}(-1) = \frac{f_0}{f_e}, \; L = f_0 + f_e + 4f$$

(A) Galilean Telescope



Here the convergent eye- piece of astronomical telescope is replaced by a divergent lens. Here M.P. = f_0 / f_e with , $L = f_0 - f_e$

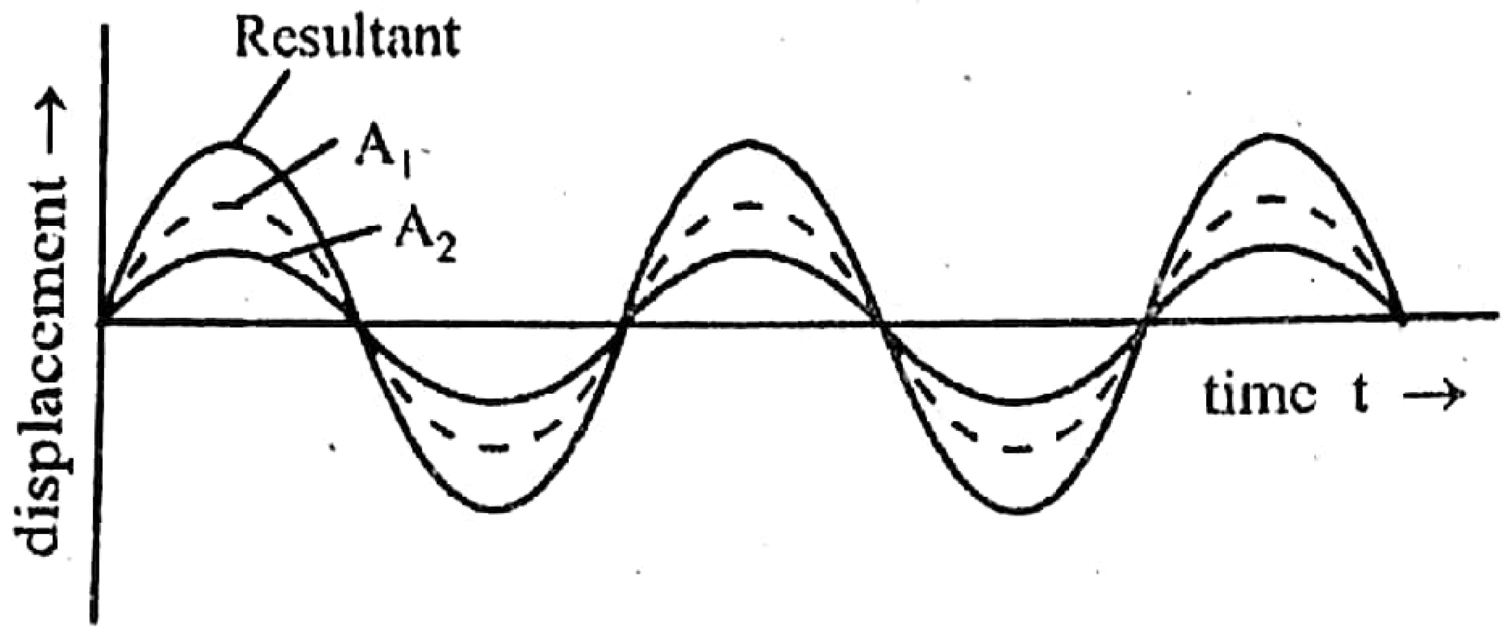
Note :

In this telescope as the intermediate image is outside the tube, the telescope cannot be used for measurements. This was not the case for all previous telescope.

$$\text{Resolving power of Telescope} = \frac{D}{1.22\lambda}$$

1. PRINCIPLE OF SUPERPOSITION

Two, or more Progressive waves can travel simultaneously in a medium without affecting the motion of one another, Therefore, the resultant displacement of each particle of the medium at any instant is equal to the vector sum of the displacements produced by the two waves separately. This principle is called principle of superposition.



Resultant displacement :

$\vec{y} = \vec{y_1} + \vec{y_2}$

Meaning of principle of superposition :

The principle of superposition means that if a number of waves are travelling in a medium, then each one travels independently as if the other waves were not present at all; the shape and other characteristics of any wave are not changed.

2. INTERFERENCE OF TWO WAVES

When two waves of same frequency travel in a medium simultaneously in the same direction then, due to their superposition, the resultant intensity at any point of the medium is different from the sum of intensities of the two waves. At some points the intensity of the resulatant wave is very large while at some other points it is very small or zero. This phenomenon is called the interference of waves.

2.1 Mathematical Interpretation of interference of two waves :

Let us consider two simple harmonic progressive waves of the same frequency travelling in the same direction. Let a_1 and a_2 be the amplitudes of the waves and ϕ the phase difference between them at any point in the medium. If the displacements due to these waves at that point be y_1 and y_2 then.

$y_1 = a_1 \sin \omega t$ —— (i)

and $y_2 = a_2 \sin (\omega t + \phi)$ —— (ii)

$\omega/2\pi$ is the frequency of each wave. By the principle of superposition, the resultant displacement at the point is given by

$y = y_1 + y_2$

$= a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$

$= a_1 \sin \omega t + a_2 \sin \omega t \cos\phi + a_2 \cos \omega t \sin \phi$

$= \sin \omega t(a_1 + a_2 \cos \phi) + \cos \omega t (a_2 \sin \phi)$

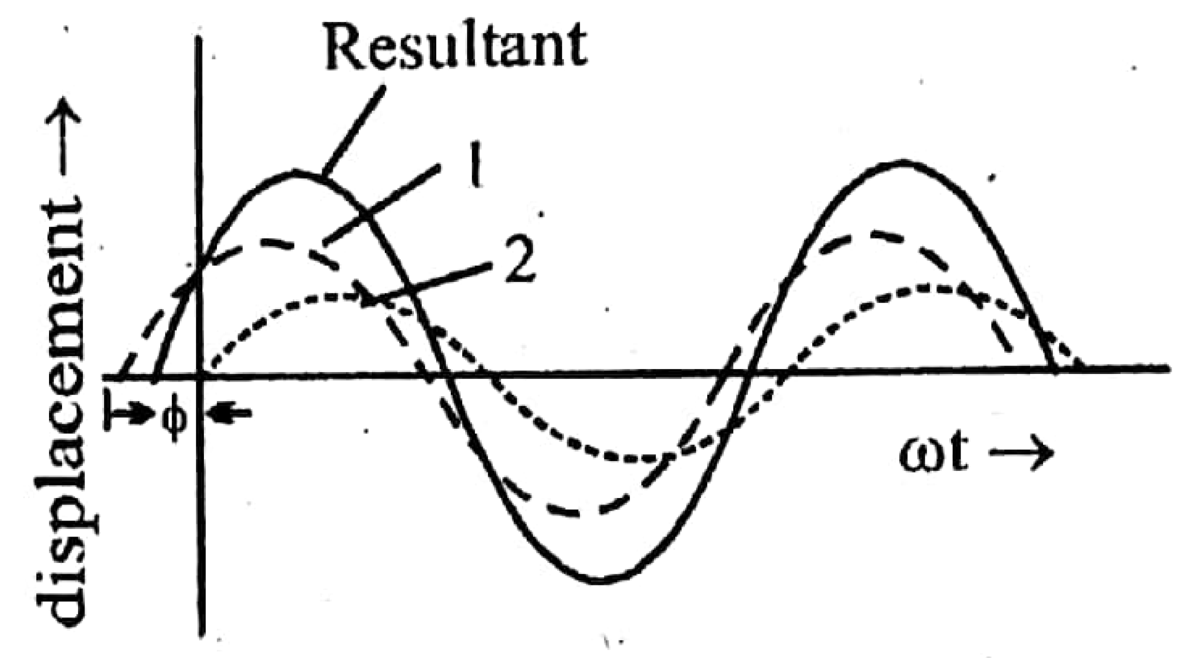
Let $a_1 + a_2 \cos \phi = R \cos \theta$ .... (iii)

and $a_2 \sin \phi = R \sin \theta$. .... (iv)

where R and θ are new constants. Then

$y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$

or $y = R \sin (\omega t + \theta)$



This equation is similar to eq. (i) and (ii). Hence the resultant displacement at that point is due to a similar wave of amplitude R. To determine R, we square eq. (iii) and (iv) and then add :

$R^2 \cos^2\theta + R^2 \sin^2\theta = (a_1 + a_2 \cos\phi)^2 + (a_2 \sin \phi)^2$

or $R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos \phi$.

The intensity is directly proportional to the square of the amplitude. Hence the resultant intensity I is given by

$I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$. and phase angle ϕ

$= \tan^{-1} \dfrac{a_2 \sin \phi}{a_1 + a_2 \cos \phi}$

Thus the resultant intensity at any point depends upon the phase difference ϕ between the two waves at that point.

Comment : if ϕ phase diff is equivalent to x path difference or Δt time difference, then remember

$\dfrac{\phi}{2\pi} = \dfrac{x}{\lambda} = \dfrac{\Delta t}{T}$

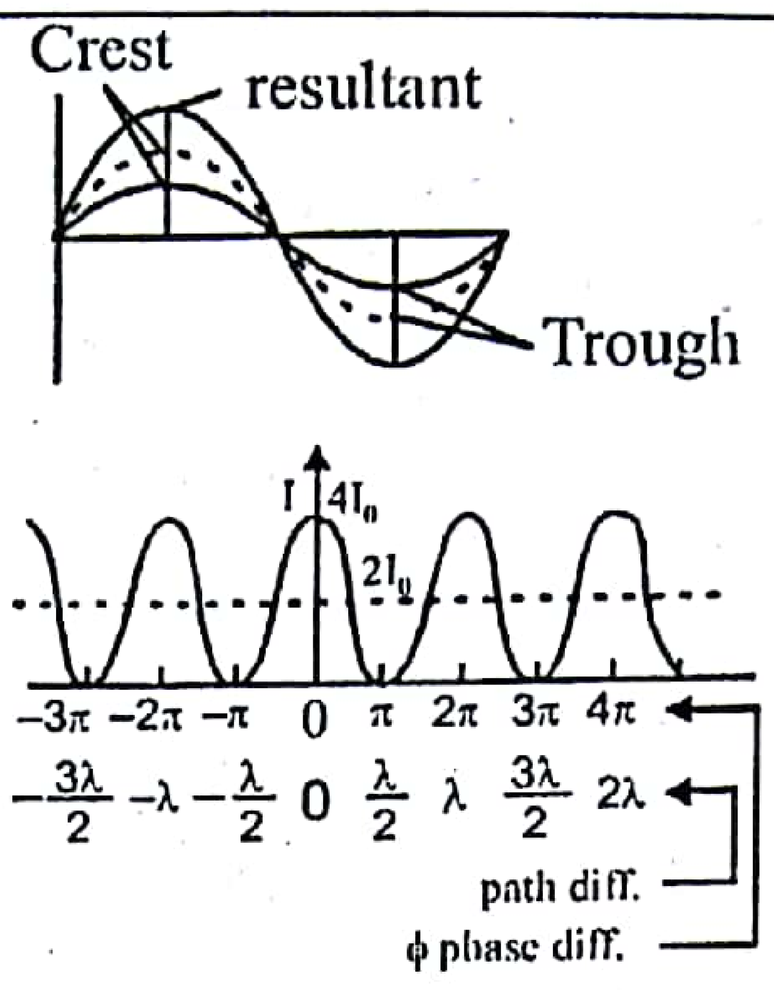
3. TWO TYPES OF INTERFERENCE

(1) Constructive (2) Destructive.

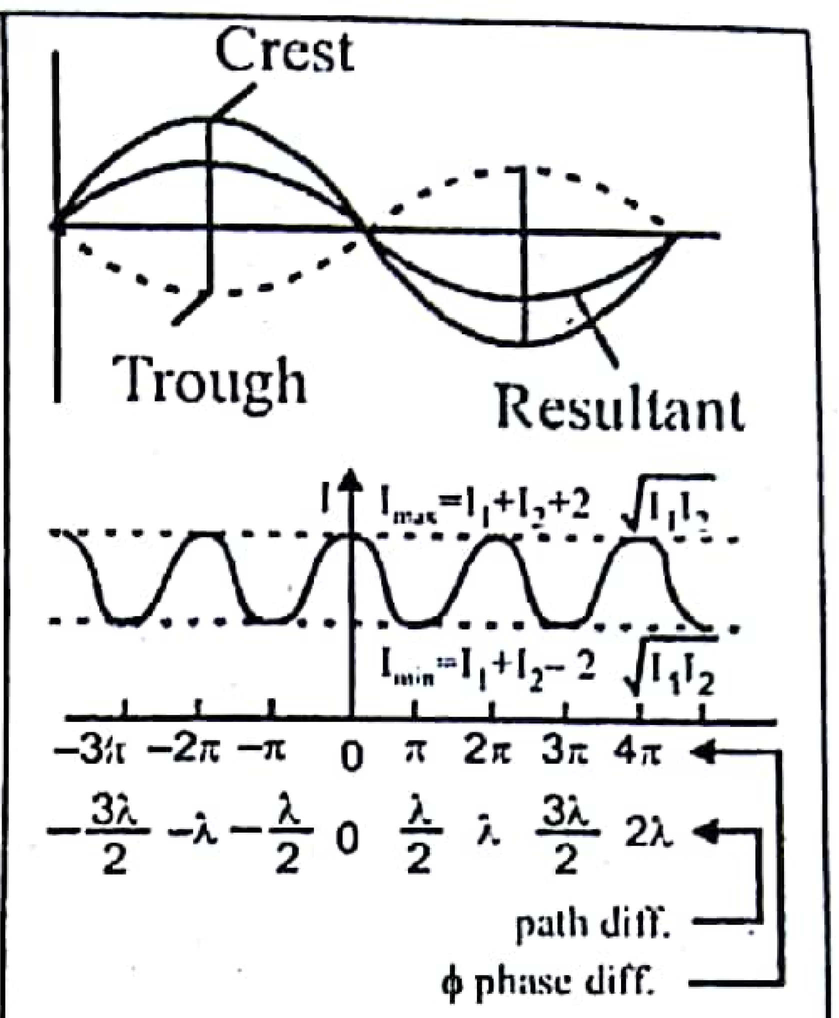
Law of conservation of energy in constructive & Destructive interference :

Energy distributes ununiformly by interference. Neither energy produces nor distroys. It changes from constructive to destructive & from destructive to constructive. Energy redistributes. It follows law of conservation of energy. comparative study of constructive & destructive interference.

CONSTRUCTIVE ($I > I_1 + I_2$)	DESTRUCTIVE ($I < I_1 + I_2$)
1. The phase difference between two waves an even multiple of π i.e. $\delta = 2n\pi$ where n = 0, 1, 2	The phase difference between two waves is an odd multiple of π. $\delta = (2n - 1)\pi$ where n = 1, 2, 3
2. The path difference between two waves is an even multiple of $\lambda/2$. $\Delta = 2n(\lambda/2)$ where n = 0,1, 2	The path difference between two waves is an odd multiple of $\lambda/2$. $\Delta = (2n - 1)\lambda/2$, where n = 1, 2
3. The time interval between two waves is even multiple of T/2 $\theta = 2n\left(\frac{T}{2}\right)$, n = 0, 1, 2	The time interval between two waves is an odd multiple of T/2, $\theta = (2n-1)T/2$, n =1, 2, 3
4. The resultant amplitude of wave is equal to the sum of amplitudes of individual waves $A = a_1 + a_2$ if $\phi = 0, 2\pi, 4\pi, ... 2n\pi$	The resultant amplitude of wave is equal to the difference of amplitudes of two waves $A = a_1 - a_2$ if $\phi = \pi, 3\pi, 5\pi....(2n-1)\pi$
5. The resultant intensity is more than the sun of intensities of individual waves $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ $= (\sqrt{I_1} + \sqrt{I_2})^2$	The resultant intensity is less than the sum of intensities due individual waves $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ $= (\sqrt{I_1} - \sqrt{I_2})^2$



Intensity of resultant wave (interfernce) pattern when the waves have equal intensities.



Intesity of reslutant wave (interference) pattern When the intensity of the two waves are not equal note that the average intensity is

$$I = \frac{I_{max} + I_{min}}{2} = I_1 + I_2$$

4. COHERENT SOURCE

The two sources of light, whose frequencies are same and the phase difference between the waves emitted by which remains constant with respect to time are defined as coherent sources.

Main points :

1. They are obtained from the same single source
2. Their state of polarisation is the same

Note :

1. Laser light is highly coherent & monochromatic
2. The light emitted by two independent sources (candles, bulbs etc.) is non coherent and interference phenomenon can not be produced by such two sources